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CHANDIGARH REGION

CLASS-XI

SUBJECT-PHYSICS

Chapter -4

Motion in Plane

INDEX

Gist of Chapter

Motion in a Plane :-

In this chapter, students will learn about Scalar and vector quantities and different types of vectors such as Position and displacement vectors, general vectors.

They will also learn about their notations. Equality of vectors, multiplication of vectors by a real number will be performed. Also addition and subtraction of vectors will be explained and rules for addition and subtraction of vectors are explained thoroughly.

Concept of relative velocity, Unit vector, resolution of a vector in a plane, rectangular components, Scalar and Vector product of vectors are also explained .

Motion in a plane, cases of uniform velocity and uniform acceleration-projectile motion, uniform circular motion topics are also explained in this chapter.

Notes and Formulae used in chapter

Introduction of Vector

Physical quantities having magnitude, direction and obeying laws of vector algebra are called vectors.

Example : Displacement, velocity, acceleration, momentum, force, impulse, weight, thrust, torque, angular momentum, angular velocity etc.

If a physical quantity has magnitude and direction both, then it does not always imply that it is a vector. For it to be a vector the third condition of obeying laws of vector algebra has to be satisfied.

Example : The physical quantity current has both magnitude and direction but is still a scalar as it disobeys the laws of vector algebra.

Types of Vector

(1) **Equal vectors :** Two vectors $\vec{A}$ and $\vec{B}$ are said to be equal when they have equal magnitudes and same direction.

(2) **Parallel vector :** Two vectors $\vec{A}$ and $\vec{B}$ are said to be parallel when

(i) Both have same direction.

(ii) One vector is scalar (positive) non-zero multiple of another vector.

(3) **Anti-parallel vectors :** Two vectors $\vec{A}$ and $\vec{B}$ are said to be anti-parallel when

(i) Both have opposite direction.

(ii) One vector is scalar non-zero negative multiple of another vector.

(4) **Collinear vectors :** When the vectors under consideration can share the same support or have a common support then the considered vectors are collinear.

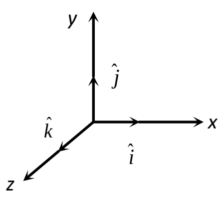
(5) **Zero vector** $(\vec{0})$ **:** A vector having zero magnitude and arbitrary direction (not known to us) is a zero vector.

(6) **Unit vector :** A vector divided by its magnitude is a unit vector. Unit vector for $\vec{A}$ is $\hat{A}$ (read as A cap or A hat).

Since, $\hat{A} = \dfrac{\vec{A}}{A} \Rightarrow \vec{A} = A\hat{A}$.

Thus, we can say that unit vector gives us the direction.

(7) **Orthogonal unit vectors** $\hat{i}, \hat{j}$ and $\hat{k}$ are called orthogonal unit vectors. These vectors must form a Right Handed Triad (It is a coordinate system such that when we Curl the fingers of right hand from x to y then we must get the direction of z along thumb). The

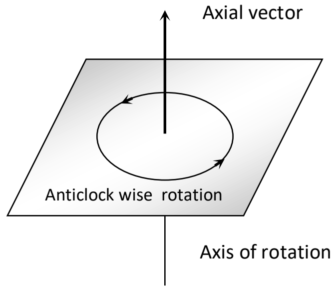


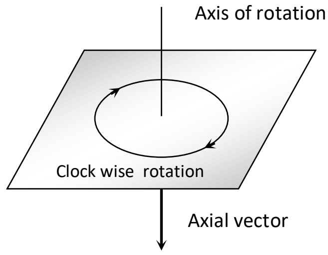
$$\hat{i} = \frac{\vec{x}}{x},\ \hat{j} = \frac{\vec{y}}{y},\ \hat{k} = \frac{\vec{z}}{z}$$

$$\therefore \quad \vec{x} = x\hat{i},\ \vec{y} = y\hat{j},\ \vec{z} = z\hat{k}$$

(8) **Polar vectors :** These have starting point or point of application . Example displacement and force *etc*.

(9) **Axial Vectors :** These represent rotational effects and are always along the axis of rotation in accordance with right hand screw rule. Angular velocity, torque and angular momentum, etc., are example of physical quantities of this type.

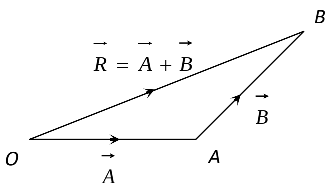




(10) **Coplanar vector** : Three (or more) vectors are called coplanar vector if they lie in the same plane. Two (free) vectors are always coplanar.

Triangle Law of Vector Addition of Two Vectors

If two non zero vectors are represented by the two sides of a triangle taken in same order then the resultant is given by the closing side of triangle in opposite order. *i.e.* $\vec{R} = \vec{A} + \vec{B}$



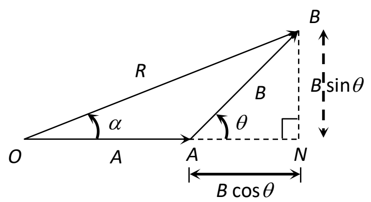
$\because \; \overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$

(1) **Magnitude of resultant vector**

In $\Delta\, ABN$, $\cos\theta = \dfrac{AN}{B}$ $\therefore\; AN = B\cos\theta$

$\sin\theta = \dfrac{BN}{B}$ $\therefore\; BN = B\sin\theta$

In ΔOBN, we have $OB^2 = ON^2 + BN^2$



$\Rightarrow\; R^2 = (A + B\cos\theta)^2 + (B\sin\theta)^2$

$\Rightarrow\; R^2 = A^2 + B^2\cos^2\theta + 2AB\cos\theta + B^2\sin^2\theta$

$\Rightarrow\; R^2 = A^2 + B^2(\cos^2\theta + \sin^2\theta) + 2AB\cos\theta$

$\Rightarrow\; R^2 = A^2 + B^2 + 2AB\cos\theta$

$\Rightarrow\; R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

(2) **Direction of resultant vectors :** If θ is angle between $\vec{A}$ and $\vec{B}$, then

$$|\vec{A}+\vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

If $\vec{R}$ makes an angle α with $\vec{A}$, then in ΔOBN,

$$\tan\alpha = \frac{BN}{ON} = \frac{BN}{OA + AN}$$

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

Parallelogram Law of Vector Addition

If two non zero vectors are represented by the two adjacent sides of a parallelogram then the resultant is given by the diagonal of the parallelogram passing through the point of intersection of the two vectors.

(1) **Magnitude**

Since, $R^2 = ON^2 + CN^2$

$$\Rightarrow R^2 = (OA + AN)^2 + CN^2$$

$$\Rightarrow R^2 = A^2 + B^2 + 2AB\cos\theta$$

$$\therefore \quad R = |\vec{R}| = |\vec{A}+\vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

* $R = A + B$ when $\theta = 0^o$

$R = A - B$ when $\theta = 180^o$

$R = \sqrt{A^2 + B^2}$ when $\theta = 90^o$

(2) **Direction**

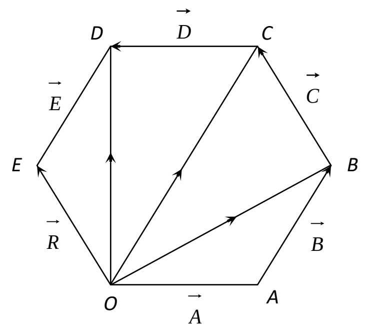
$$\tan\beta = \frac{CN}{ON} = \frac{B\sin\theta}{A + B\cos\theta}$$

Polygon Law of Vector Addition

If a number of non zero vectors are represented by the (*n* – 1) sides of an *n*-sided polygon then the resultant is given by the closing side or the n^{th} side of the polygon taken in opposite order. So,

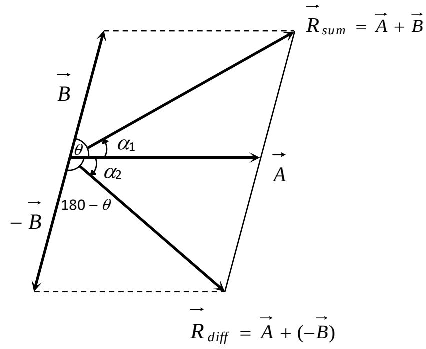
$$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D} + \vec{E}$$

$$\overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} = \overrightarrow{OE}$$



- ❑ Resultant of two unequal vectors can not be zero.
- ❑ Resultant of three co-planar vectors may or may not be zero
- ❑ Resultant of three non co- planar vectors can not be zero.

Subtraction of vectors



Since, $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$ and

$$|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$\Rightarrow |\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos(180^o - \theta)}$$

Since, $\cos(180 - \theta) = -\cos\theta$

$$\Rightarrow |\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

$$\tan\alpha_1 = \frac{B\sin\theta}{A + B\cos\theta}$$

and $\tan\alpha_2 = \dfrac{B\sin(180 - \theta)}{A + B\cos(180 - \theta)}$

But $\sin(180 - \theta) = \sin\theta$ and $\cos(180 - \theta) = -\cos\theta$

$$\Rightarrow \tan\alpha_2 = \frac{B\sin\theta}{A - B\cos\theta}$$

Resolution of Vector Into Components

Consider a vector $\vec{R}$ in X-Y plane as shown in fig. If we draw orthogonal vectors $\vec{R}_x$ and $\vec{R}_y$ along x and y axes respectively, by law of vector addition, $\vec{R} = \vec{R}_x + \vec{R}_y$

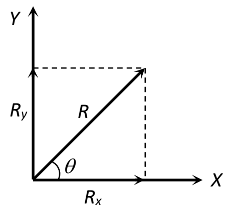


Fig. 0.8

Now as for any vector $\vec{A} = A\hat{n}$ so, $\vec{R}_x = \hat{i}R_x$ and $\vec{R}_y = \hat{j}R_y$

so $\vec{R} = \hat{i}R_x + \hat{j}R_y$ …(i)

But from figure $R_x = R\cos\theta$ …(ii)

and $R_y = R\sin\theta$ …(iii)

Since R and θ are usually known, Equation (ii) and (iii) give the magnitude of the components of $\vec{R}$ along x and y-axes respectively.

Here it is worthy to note once a vector is resolved into its components, the components themselves can be used to specify the vector as

(1) The magnitude of the vector $\vec{R}$ is obtained by squaring and adding equation (ii) and (iii), *i.e.*

$$R = \sqrt{R_x^2 + R_y^2}$$

(2) The direction of the vector $\vec{R}$ is obtained by dividing equation (iii) by (ii), *i.e.*

$\tan\theta = (R_y / R_x)$ or $\theta = \tan^{-1}(R_y / R_x)$

Rectangular Components of 3-D Vector

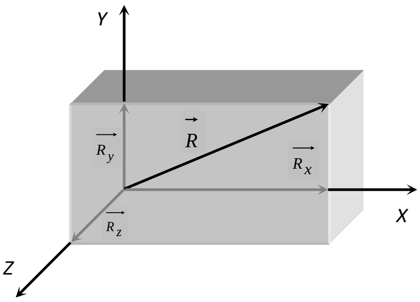
$$\vec{R} = \vec{R}_x + \vec{R}_y + \vec{R}_z \text{ or } \vec{R} = R_x\hat{i} + R_y\hat{j} + R_z\hat{k}$$

If $\vec{R}$ makes an angle α with x axis, β with y axis and γ with z axis, then

$$\Rightarrow \cos\alpha = \frac{R_x}{R} = \frac{R_x}{\sqrt{R_x^2 + R_y^2 + R_z^2}} = l$$

$$\Rightarrow \cos\beta = \frac{R_y}{R} = \frac{R_y}{\sqrt{R_x^2 + R_y^2 + R_z^2}} = m$$

$$\Rightarrow \cos\gamma = \frac{R_z}{R} = \frac{R_z}{\sqrt{R_x^2 + R_y^2 + R_z^2}} = n$$



Where l, m, n are called Direction Cosines of the vector $\vec{R}$ and

$$l^2 + m^2 + n^2 = \cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{R_x^2 + R_y^2 + R_z^2}{R_x^2 + R_y^2 + R_z^2} = 1$$

- When a point P have coordinate (x, y, z) then its position vector $\overrightarrow{OP} = x\hat{i} + y\hat{j} + z\hat{k}$
- When a particle moves from point (x_1, y_1, z_1) to (x_2, y_2, z_2) then its displacement vector

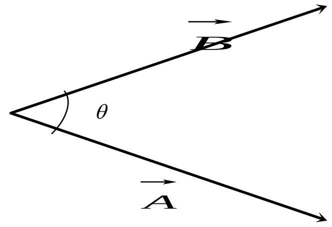
$$\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

Scalar Product of Two Vectors

(1) **Definition :** The scalar product (or dot product) of two vectors is defined as the product of the magnitude of two vectors with cosine of angle between them.

Thus if there are two vectors $\vec{A}$ and $\vec{B}$ having angle θ between them, then their scalar product written as $\vec{A}.\vec{B}$ is defined as $\vec{A}.\vec{B} = AB\cos\theta$

(2) **Properties :** (i) It is always a scalar which is positive if angle between the vectors is acute (*i.e.*, < 90°) and negative if angle between them is obtuse (*i.e.* 90°< θ < 180°).



(ii) It is commutative, *i.e.* $\vec{A}.\vec{B} = \vec{B}.\vec{A}$

(iii) It is distributive, *i.e.* $\vec{A}.(\vec{B}+\vec{C}) = \vec{A}.\vec{B} + \vec{A}.\vec{C}$

(iv) As by definition $\vec{A}.\vec{B} = AB\cos\theta$

The angle between the vectors $\theta = \cos^{-1}\left[\frac{\vec{A}.\vec{B}}{AB}\right]$

(v) Scalar product of two vectors will be maximum when $\cos\theta = \max = 1$, *i.e.* $\theta = 0^o$, *i.e.*, vectors are parallel

$$(\vec{A}.\vec{B})_{\max} = AB$$

(vi) Scalar product of two vectors will be minimum when $|\cos\theta| = \min = 0$, *i.e.* $\theta = 90^o$

$(\vec{A}.\vec{B})_{\min} = 0$

i.e. if the scalar product of two nonzero vectors vanishes the vectors are orthogonal.

(vii) The scalar product of a vector by itself is termed as self dot product and is given by $(\vec{A})^2 = \vec{A}.\vec{A} = AA\cos\theta = A^2$

i.e. $A = \sqrt{\vec{A}.\vec{A}}$

(viii) In case of unit vector $\hat{n}$

$\hat{n}.\hat{n} = 1\times 1\times \cos 0 = 1$ so $\hat{n}.\hat{n} = \hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$

(ix) In case of orthogonal unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$, $\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 1\times 1\cos 90° = 0$

(x) In terms of components

$\vec{A}.\vec{B} = (\hat{i}A_x + \hat{j}A_y + \hat{k}A_z).(\hat{i}B_x + \hat{j}B_y + \hat{k}B_z) = [A_xB_x + A_yB_y + A_ZB_z]$

(3) **Example :** (i) Work W : In physics for constant force work is defined as, $W = Fs\cos\theta$ ...(i)

But by definition of scalar product of two vectors, $\vec{F}.\vec{s} = Fs\cos\theta$ ...(ii)

So from eq^n (i) and (ii) $W = \vec{F}.\vec{s}$ *i.e.* work is the scalar product of force with displacement.

(ii) Power P :

As $W = \vec{F}.\vec{s}$ or $\frac{dW}{dt} = \vec{F}.\frac{d\vec{s}}{dt}$ [As $\vec{F}$ is constant]

or $P = \vec{F}.\vec{v}$ *i.e.*, power is the scalar product of force with velocity. $\left[\text{As}\frac{dW}{dt} = P \text{ and } \frac{d\vec{s}}{dt} = \vec{v}\right]$

(iv) Potential energy of a dipole U : If an electric dipole of moment $\vec{p}$ is situated in an electric field $\vec{E}$ or a magnetic dipole of moment $\vec{M}$ in a field of induction $\vec{B}$, the potential energy of the dipole is given by :

$U_E = -\vec{p}.\vec{E}$ and $U_B = -\vec{M}.\vec{B}$

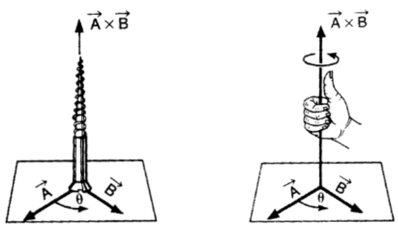
Vector Product of Two Vectors

(1) **Definition** : The vector product or cross product of two vectors is defined as a vector having a magnitude equal to the product of the magnitudes of two vectors with the sine of angle between them, and direction perpendicular to the plane containing the two vectors in accordance with right hand screw rule.

$\vec{C} = \vec{A}\times\vec{B}$

Thus, if $\vec{A}$ and $\vec{B}$ are two vectors, then their vector product written as $\vec{A}\times\vec{B}$ is a vector $\vec{C}$ defined by

$\vec{C} = \vec{A}\times\vec{B} = AB\sin\theta\,\hat{n}$



The direction of $\vec{A}\times\vec{B}$, *i.e.* $\vec{C}$ is perpendicular to the plane containing vectors $\vec{A}$ and $\vec{B}$ and in the sense of advance of a right handed screw rotated from $\vec{A}$ (first vector) to $\vec{B}$ (second vector) through the smaller angle between them. Thus, if a right handed screw whose axis is perpendicular to the plane framed by $\vec{A}$ and $\vec{B}$ is rotated from $\vec{A}$ to $\vec{B}$ through the smaller angle between them, then the direction of advancement of the screw gives the direction of $\vec{A}\times\vec{B}$ *i.e.* $\vec{C}$

(2) **Properties**

(i) Vector product of any two vectors is always a vector perpendicular to the plane containing these two vectors, *i.e.*, orthogonal to both the vectors $\vec{A}$ and $\vec{B}$, though the vectors $\vec{A}$ and $\vec{B}$ may or may not be orthogonal.

(ii) Vector product of two vectors is not commutative, *i.e.*, $\vec{A}\times\vec{B}\neq\vec{B}\times\vec{A}$ [but $=-\vec{B}\times\vec{A}$]

Here it is worthy to note that

$|\vec{A}\times\vec{B}| = |\vec{B}\times\vec{A}| = AB\sin\theta$

i.e. in case of vector $\vec{A}\times\vec{B}$ and $\vec{B}\times\vec{A}$ magnitudes are equal but directions are opposite.

(iii) The vector product is distributive when the order of the vectors is strictly maintained, *i.e.*

$$\vec{A}\times(\vec{B}+\vec{C})=\vec{A}\times\vec{B}+\vec{A}\times\vec{C}$$

(iv) The vector product of two vectors will be maximum when $\sin\theta = \max = 1$, *i.e.*, $\theta = 90^o$

$[\vec{A}\times\vec{B}]_{\max} = AB\,\hat{n}$

i.e. vector product is maximum if the vectors are orthogonal.

(v) The vector product of two non- zero vectors will be minimum when $|\sin\theta| =$ minimum = 0, *i.e.*, $\theta = 0^o$ or 180^o

$[\vec{A}\times\vec{B}]_{\min} = 0$

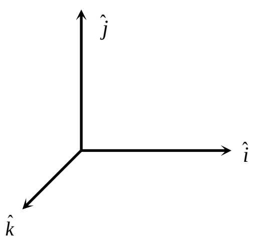
i.e. if the vector product of two non-zero vectors vanishes, the vectors are collinear.

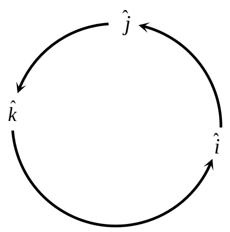
(vi) The self cross product, *i.e.*, product of a vector by itself vanishes, *i.e.*, is null vector

$\vec{A}\times\vec{A} = AA\sin 0^o\,\hat{n} = \vec{0}$

(vii) In case of unit vector $\hat{n}\times\hat{n}=\vec{0}$ so that $\hat{i}\times\hat{i}=\hat{j}\times\hat{j}=\hat{k}\times\hat{k}=\vec{0}$

(viii) In case of orthogonal unit vectors, $\hat{i},\hat{j},\hat{k}$ in accordance with right hand screw rule :





$\hat{i}\times\hat{j}=\hat{k},\ \hat{j}\times\hat{k}=\hat{i}$ and $\hat{k}\times\hat{i}=\hat{j}$

And as cross product is not commutative,

$\hat{j}\times\hat{i}=-\hat{k}$, $\hat{k}\times\hat{j}=-\hat{i}$ and $\hat{i}\times\hat{k}=-\hat{j}$

(x) In terms of components

$$\vec{A}\times\vec{B}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}$$

$$=\hat{i}(A_yB_z-A_zB_y)+\hat{j}(A_zB_x-A_xB_z)+\hat{k}(A_xB_y-A_yB_x)$$

(3) **Example :** Since vector product of two vectors is a vector, vector physical quantities (particularly representing rotational effects) like torque, angular momentum, velocity and force on a moving charge in a magnetic field and can be expressed as the vector product of two vectors. It is well – established in physics that :

(i) Torque $\vec{\tau}=\vec{r}\times\vec{F}$

(ii) Angular momentum $\vec{L}=\vec{r}\times\vec{p}$

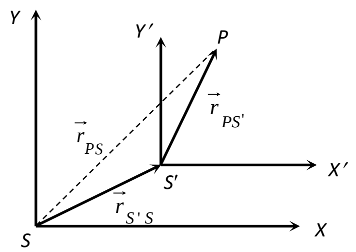
(iii) Velocity $\vec{v}=\vec{\omega}\times\vec{r}$

(iv) Force on a charged particle q moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ is given by $\vec{F}=q(\vec{v}\times\vec{B})$

(v) Torque on a dipole in a field $\vec{\tau_E}=\vec{p}\times\vec{E}$ and $\vec{\tau_B}=\vec{M}\times\vec{B}$

Relative Velocity

(1) **Introduction :** When we consider the motion of a particle, we assume a fixed point relative to which the given particle is in motion. For example, if we say that water is flowing or wind is blowing or a person is running with a speed *v*, we mean that these all are relative to the earth (which we have assumed to be fixed).



Now to find the velocity of a moving object relative to another moving object, consider a particle *P* whose position relative to frame *S* is $\vec{r}_{PS}$ while relative to S' is $\vec{r}_{PS'}$.

If the position of frames S' relative to *S* at any time is $\vec{r}_{S'S}$ then from figure, $\vec{r}_{PS}=\vec{r}_{PS'}+\vec{r}_{S'S}$

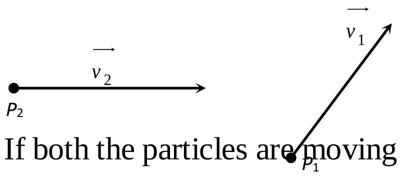
Differentiating this equation with respect to time

$$\frac{d\vec{r}_{PS}}{dt}=\frac{d\vec{r}_{PS'}}{dt}+\frac{d\vec{r}_{S'S}}{dt}$$

or $\vec{v}_{PS}=\vec{v}_{PS'}+\vec{v}_{S'S}$ [as $\vec{v}=d\vec{r}/dt$]

or $\vec{v}_{PS'}=\vec{v}_{PS}-\vec{v}_{S'S}$

(2) **General Formula :** The relative velocity of a particle P_1 moving with velocity $\vec{v}_1$ with respect to another particle P_2 moving with velocity $\vec{v}_2$ is given by, $\vec{v}_{r_{12}} = \vec{v}_1 - \vec{v}_2$



(i) If both the particles are moving in the same direction then :

$\upsilon_{r_{12}} = \upsilon_1 - \upsilon_2$

(ii) If the two particles are moving in the opposite direction, then :

$\upsilon_{r_{12}} = \upsilon_1 + \upsilon_2$

(iii) If the two particles are moving in the mutually perpendicular directions, then:

$\upsilon_{r_{12}} = \sqrt{\upsilon_1^2 + \upsilon_2^2}$

(iv) If the angle between $\vec{\upsilon}_1$ and $\vec{\upsilon}_2$ be θ, then $\upsilon_{r_{12}} = \left[\upsilon_1^2 + \upsilon_2^2 - 2\upsilon_1\upsilon_2 \cos\theta\right]^{1/2}$.

(3) **Relative velocity of satellite :** If a satellite is moving in equatorial plane with velocity $\vec{v}_s$ and a point on the surface of earth with $\vec{v}_e$ relative to the centre of earth, the velocity of satellite relative to the surface of earth

$\vec{v}_{se} = \vec{v}_s - \vec{v}_e$

So if the satellite moves form west to east (in the direction of rotation of earth on its axis) its velocity relative to earth's surface will be $v_{se} = v_s - v_e$

And if the satellite moves from east to west, *i.e.*, opposite to the motion of earth,

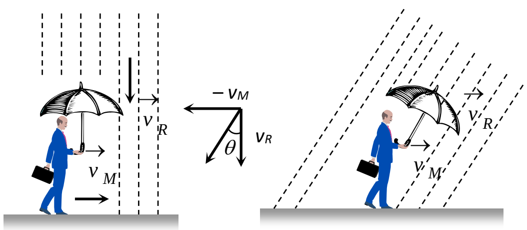
$v_{se} = v_s - (-v_e) = v_s + v_e$

(4) **Relative velocity of rain :** If rain is falling vertically with a velocity $\vec{v}_R$ and an observer is moving horizontally with speed $\vec{v}_M$ the velocity of rain relative to observer will be $\vec{v}_{RM} = \vec{v}_R - \vec{v}_M$

which by law of vector addition has magnitude

$v_{RM} = \sqrt{v_R^2 + v_M^2}$

direction $\theta = \tan^{-1}(v_M / v_R)$ with the vertical as shown in fig.



(5) **Relative velocity of swimmer :** If a man can swim relative to water with velocity $\vec{v}$ and water is flowing relative to ground with velocity $\vec{v}_R$ velocity of man relative to ground $\vec{v}_M$ will be given by:

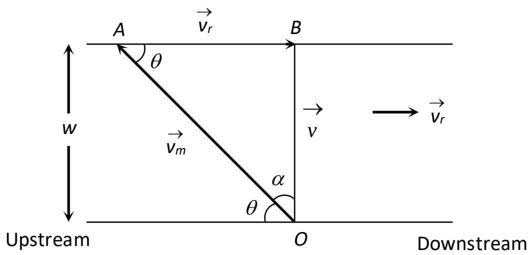
$\vec{v} = \vec{v}_M - \vec{v}_R$, *i.e.*, $\vec{v}_M = \vec{v} + \vec{v}_R$

So if the swimming is in the direction of flow of water, $v_M = v + v_R$

And if the swimming is opposite to the flow of water, $v_M = v - v_R$

(6) **Crossing the river :** Suppose, the river is flowing with velocity $\vec{\upsilon}_r$. A man can swim in still water with velocity $\vec{\upsilon}_m$. He is standing on one bank of the river and wants to cross the river, two cases arise.

(i) To cross the river over shortest distance : That is to cross the river straight, the man should swim making angle θ with the upstream as shown.



Here *OAB* is the triangle of vectors, in which $\vec{OA} = \vec{v}_m, \vec{AB} = \vec{\upsilon}_r$. Their resultant is given by $\vec{OB} = \vec{\upsilon}$. The direction of swimming makes angle θ with upstream. From the triangle *OBA*, we find,

$\cos\,\theta = \dfrac{\upsilon_r}{\upsilon_m}$ Also $\sin\alpha = \dfrac{\upsilon_r}{\upsilon_m}$

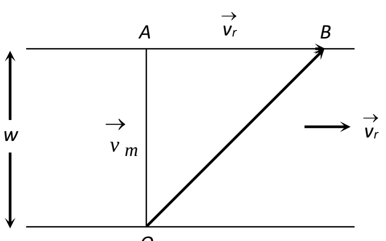
Where α is the angle made by the direction of swimming with the shortest distance (*OB*) across the river.

Time taken to cross the river : If *w* be the width of the river, then time taken to cross the river will be given by

$$t_1 = \frac{w}{\upsilon} = \frac{w}{\sqrt{\upsilon_m^2 - \upsilon_r^2}}$$

(ii) To cross the river in shortest possible time : The man should swim perpendicular to the bank.

The time taken to cross the river will be: $t_2 = \dfrac{w}{\upsilon_m}$



In this case, the man will touch the opposite bank at a distance *AB* down stream. This distance will be given by:

$$AB = \upsilon_r t_2 = \upsilon_r \frac{w}{\upsilon_m} \quad \text{or} \quad AB = \frac{\upsilon_r}{\upsilon_m} w$$

Motion In Two Dimension

The motion of an object is called two dimensional, if two of the three co-ordinates required to specify the position of the object in space, change *w.r.t* time.

In such a motion, the object moves in a plane. For example, a billiard ball moving over the billiard table, an insect crawling over the floor of a room, earth revolving around the sun *etc*.

Two special cases of motion in two dimension are

1. Projectile motion 2. Circular motion

1. Projectile motion

A body which is in flight through the atmosphere under the effect of gravity alone and is not being propelled by any fuel is called projectile.

Example:

(i) A bomb released from an aeroplane in level flight

(ii) A bullet fired from a gun

(iii) An arrow released from bow

(iv) A Javelin thrown by an athlete

Assumptions of Projectile Motion

(1) There is no resistance due to air.

(2) The effect due to curvature of earth is negligible.

(3) The effect due to rotation of earth is negligible.

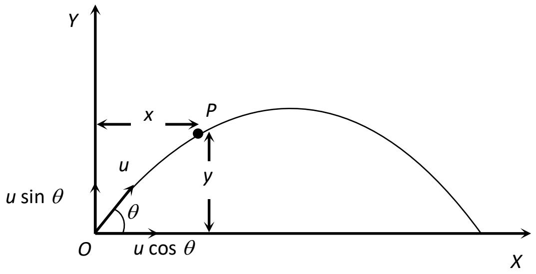
(4) For all points of the trajectory, the acceleration due to gravity 'g' is constant in magnitude and direction

Oblique Projectile

In projectile motion, horizontal component of velocity ($u \cos\theta$), acceleration (g) and mechanical energy remains constant while, speed, velocity, vertical component of velocity (u

sin θ), momentum, kinetic energy and potential energy all changes. Velocity, and KE are maximum at the point of projection while minimum (but not zero) at highest point.

(1) **Equation of trajectory :** A projectile is thrown with velocity u at an angle θ with the horizontal. The velocity u can be resolved into two rectangular components.



$v\cos\theta$ component along X–axis and $u\sin\theta$ component along Y–axis.

For horizontal motion $x = u\cos\theta \times t \;\Rightarrow\; t = \dfrac{x}{u\cos\theta}$ … (i)

For vertical motion $y = (u\sin\theta)t - \frac{1}{2}gt^2$ …(ii)

From equation (i) and (ii)

$$y = u\sin\theta\left(\frac{x}{u\cos\theta}\right) - \frac{1}{2}g\left(\frac{x^2}{u^2\cos^2\theta}\right)$$

$$y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta}$$

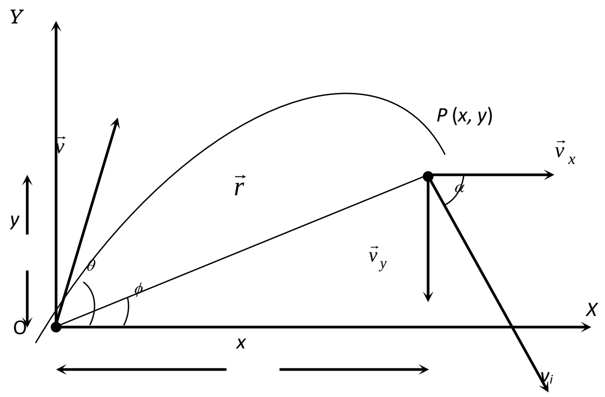
This equation shows that the trajectory of projectile is parabolic because it is similar to equation of parabola

$y = ax - bx^2$

❑ Equation of oblique projectile also can be written as

$y = x\tan\theta\left[1 - \dfrac{x}{R}\right]$ (where R = horizontal range = $\dfrac{u^2\sin 2\theta}{g}$)

(2) **Displacement of projectile ($\vec{r}$) :** Let the particle acquires a position P having the coordinates (x, y) just after time t from the instant of projection. The corresponding position vector of the particle at time t is $\vec{r}$ as shown in the figure.



$\vec{r} = x\hat{i} + y\hat{j}$ ...(i)

The horizontal distance covered during time t is given as

$x = v_x\, t \Rightarrow x = u\cos\theta\ t$ ...(ii)

The vertical velocity of the particle at time t is given as

$v_y = (v_0)_y - gt,$ ...(iii)

Now the vertical displacement y is given as

$y = u\sin\theta\, t - 1/2\, gt^2$ ...(iv)

Putting the values of x and y from equation (ii) and equation (iv) in equation (i) we obtain the position vector at any time t as

$$\vec{r} = (u\cos\theta)t\,\hat{i} + \left((u\sin\theta)t - \frac{1}{2}gt^2\right)\hat{j}$$

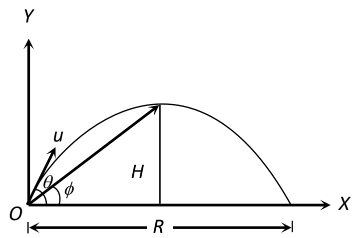
$$\Rightarrow r = \sqrt{(u\,t\cos\theta)^2 + \left((u\,t\sin\theta) - \frac{1}{2}gt^2\right)^2}$$

$$r = u\,t\sqrt{1 + \left(\frac{gt}{2u}\right)^2 - \frac{gt\sin\theta}{u}} \text{ and } \phi = \tan^{-1}(y/x)$$

$$= \tan^{-1}\left(\frac{ut\sin\theta - \frac{1}{2}gt^2}{(u\,t\cos\theta)}\right) \text{ or } \phi = \tan^{-1}\left(\frac{2u\sin\theta - gt}{2u\cos\theta}\right)$$

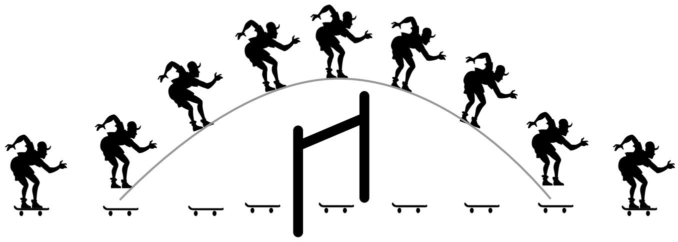
- The angle of elevation ϕ of the highest point of the projectile and the angle of projection θ are related to each other as

$$\tan\phi = \frac{1}{2}\tan\theta$$



(3) **Instantaneous velocity *v* :** In projectile motion, vertical component of velocity changes but horizontal component of velocity remains always constant.

Example : When a man jumps over the hurdle leaving behind its skateboard then vertical component of his velocity is changing, but not the horizontal component which matches with the skateboard velocity.As a result, the skateboard stays underneath him, allowing him to land on it.



Let v_i be the instantaneous velocity of projectile at time t, direction of this velocity is along the tangent to the trajectory at point P.

$\vec{v}_i = v_x i + v_y \hat{j} \Rightarrow v_i = \sqrt{v_x^2 + v_y^2}$

$= \sqrt{u^2 \cos^2\theta + (u\sin\theta - gt)^2}$

$v_i = \sqrt{u^2 + g^2t^2 - 2u\, gt\sin\theta}$

Direction of instantaneous velocity $\tan\alpha = \dfrac{v_y}{v_x} = \dfrac{u\sin\theta - gt}{u\cos\theta}$

or $\alpha = \tan^{-1}\left[\tan\theta - \dfrac{gt}{u}\sec\theta\right]$

(4) **Change in velocity :** Initial velocity (at projection point) $\vec{u}_i = u\cos\theta\hat{i} + u\sin\theta\hat{j}$

Final velocity (at highest point) $\vec{u}_f = u\cos\theta\hat{i} + 0\hat{j}$

(i) Change in velocity (Between projection point and highest point) $\Delta\vec{u} = \vec{u}_f - \vec{u}_i = -u\sin\theta\hat{j}$

When body reaches the ground after completing its motion then final velocity $\vec{u}_f = u\cos\theta\hat{i} - u\sin\theta\hat{j}$

(ii) Change in velocity (Between complete projectile motion) $\Delta\vec{u} = u_f - u_i = -2u\sin\theta\hat{i}$

(5) **Time of flight :** The total time taken by the projectile to go up and come down to the same level from which it was projected is called time of flight.

For vertical upward motion $0 = u\sin\theta - gt$

$\Rightarrow t = (u\sin\theta/g)$

Now as time taken to go up is equal to the time taken to come down so

Time of flight $T = 2t = \dfrac{2u\sin\theta}{g}$

(i) Time of flight can also be expressed as : $T = \dfrac{2.u_y}{g}$ (where u_y is the vertical component of initial velocity).

(ii) For complementary angles of projection θ and $90^o - \theta$

(a) Ratio of time of flight $= \dfrac{T_1}{T_2} = \dfrac{2u\sin\theta/g}{2u\sin(90-\theta)/g}$

$= \tan\theta \Rightarrow \dfrac{T_1}{T_2} = \tan\theta$

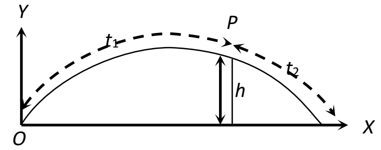
(b) Multiplication of time of flight = $T_1T_2 = \dfrac{2u\sin\theta}{g}\dfrac{2u\cos\theta}{g}$

$\Rightarrow T_1T_2 = \dfrac{2R}{g}$

(iii) If t_1 is the time taken by projectile to rise upto point p and t_2 is the time taken in falling from point p to ground level then $t_1 + t_2 = \dfrac{2u\sin\theta}{g} =$ time of flight or $u\sin\theta = \dfrac{g(t_1+t_2)}{2}$

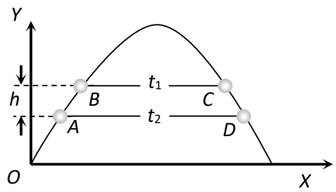
and height of the point p is given by $h = u\sin\theta\, t_1 - \dfrac{1}{2}gt_1^2$

$h = g\dfrac{(t_1+t_2)}{2}t_1 - \dfrac{1}{2}gt_1^2$



by solving $h = \frac{g t_1 t_2}{2}$

(iv) If B and C are at the same level on trajectory and the time difference between these two points is t_1, similarly A and D are also at the same level and the time difference between these two positions is t_2 then



$$t_2^2 - t_1^2 = \frac{8h}{g}$$

(6)Horizontal range :

It is the horizontal distance travelled by a body during the time of flight.

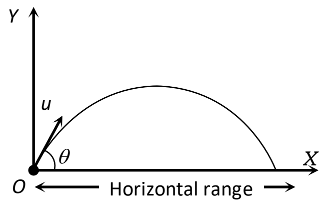
So by using second equation of motion in x-direction

$$R = u\cos\theta \times T$$

$$= u\cos\theta \times (2u\sin\theta / g)$$

$$= \frac{u^2 \sin 2\theta}{g}$$

$$R = \frac{u^2 \sin 2\theta}{g}$$



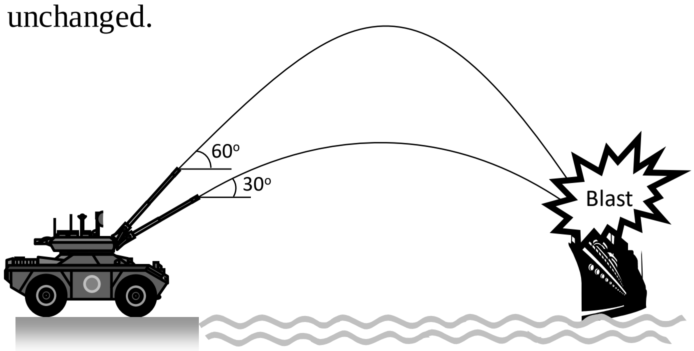
(i) Range of projectile can also be expressed as :

$R = u\cos\theta \times T = u\cos\theta \frac{2u\sin\theta}{g}$

$$= \frac{2u\cos\theta\; u\sin\theta}{g} = \frac{2u_x u_y}{g}$$

$\therefore\ R = \frac{2u_x u_y}{g}$ (where u_x and u_y are the horizontal and vertical component of initial velocity)

(ii) If angle of projection is changed from θ to $\theta' = (90 - \theta)$ then range remains unchanged.



$$R' = \frac{u^2 \sin 2\theta'}{g} = \frac{u^2 \sin[2(90^o - \theta)]}{g} = \frac{u^2 \sin 2\theta}{g} = R$$

So a projectile has same range at angles of projection θ and $(90 - \theta)$, though time of flight, maximum height and trajectories are different.

These angles θ and $90^o - \theta$ are called complementary angles of projection and for complementary angles of projection, ratio of range $\frac{R_1}{R_2} = \frac{u^2 \sin 2\theta / g}{u^2 \sin[2(90^o - \theta)]/g} = 1 \Rightarrow \frac{R_1}{R_2} = 1$

(iii) For angle of projection $\theta_1 = (45 - \alpha)$ and $\theta_2 = (45 + \alpha)$, range will be same and equal to $u^2 \cos 2\alpha/g$.

θ_1 and θ_2 are also the complementary angles.

(iv) Maximum range : For range to be maximum

$$\frac{dR}{d\theta} = 0 \Rightarrow \frac{d}{d\theta}\left[\frac{u^2 \sin 2\theta}{g}\right] = 0$$

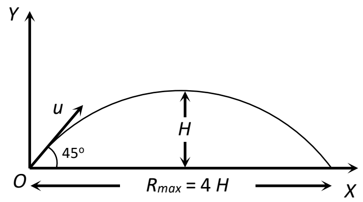
$\Rightarrow \cos 2\theta = 0$ *i.e.* $2\theta = 90^o \Rightarrow \theta = 45^o$

and $R_{max} = (u^2/g)$

i.e., a projectile will have maximum range when it is projected at an angle of 45^o to the horizontal and the maximum range will be (u^2/g).

When the range is maximum, the height H reached by the projectile

$$H = \frac{u^2 \sin^2 \theta}{2g} = \frac{u^2 \sin^2 45}{2g} = \frac{u^2}{4g} = \frac{R_{\max}}{4}$$



i.e., if a person can throw a projectile to a maximum distance R_{max}, The maximum height during the flight to which it will rise is $\left(\frac{R_{\max}}{4}\right)$.

(v) Relation between horizontal range and maximum height : $R = \frac{u^2 \sin 2\theta}{g}$ and $H = \frac{u^2 \sin^2 \theta}{2g}$

$$\therefore \quad \frac{R}{H} = \frac{u^2 \sin 2\theta / g}{u^2 \sin^2 \theta / 2g} = 4 \cot \theta \Rightarrow R = 4H \cot \theta$$

(vi) If in case of projectile motion range R is n times the maximum height H

i.e. $R = nH \Rightarrow \frac{u^2 \sin 2\theta}{g} = n \frac{u^2 \sin^2 \theta}{2g}$

$\Rightarrow \tan \theta = [4/n]$ or $\theta = \tan^{-1}[4/n]$

The angle of projection is given by $\theta = \tan^{-1}[4/n]$

❑If $R = H$ then $\theta = \tan^{-1}(4)$ or $\theta = 76^o$.

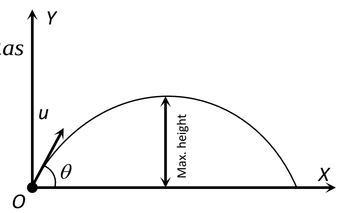
If $R = 4H$ then $\theta = \tan^{-1}(1)$ or $\theta = 45^o$.

(7)**Maximum height :** It is the maximum height from the point of projection, a projectile can reach.

So, by using $v^2 = u^2 + 2as$

$$0 = (u \sin \theta)^2 - 2gH$$

$$H = \frac{u^2 \sin^2 \theta}{2g}$$



(i) Maximum height can also be expressed as

$H = \frac{u_y^2}{2g}$ (where u_y is the vertical component of initial velocity).

(ii) $H_{max} = \frac{u^2}{2g}$ (when $\sin^2\theta$ = max = 1 *i.e.,* $\theta = 90^o$)

i.e., for maximum height body should be projected vertically upward. So it falls back to the point of projection after reaching the maximum height.

(iii) For complementary angles of projection θ and $90^o - \theta$

Ratio of maximum height

$$= \frac{H_1}{H_2} = \frac{u^2 \sin^2\theta / 2g}{u^2 \sin^2(90^o - \theta)/2g} = \frac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta$$

$$\therefore \quad \frac{H_1}{H_2} = \tan^2\theta .$$

Circular Motion

Circular motion is another example of motion in two dimensions. To create circular motion in a body it must be given some initial velocity and a force must then act on the body which is always directed at right angles to instantaneous velocity.

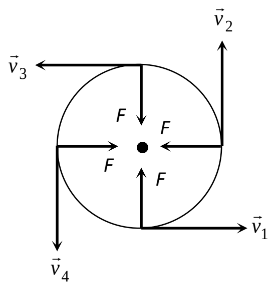


Fig : 3.22

Since this force is always at right angles to the displacement therefore no work is done by the force on the particle. Hence, its kinetic energy and thus speed is unaffected. But due to simultaneous action of the force and the velocity the particle follows resultant path, which in this case is a circle. Circular motion can be classified into two types – Uniform circular motion and non-uniform circular motion.

Variables of Circular Motion

(1) **Displacement and distance :** When particle moves in a circular path describing an angle θ during time *t* (as shown in the figure) from the position *A* to the position *B*, we see that the magnitude of the position vector $\vec{r}$ (that is equal to the radius of the circle) remains constant. *i.e.,* $|\vec{r}_1| = |\vec{r}_2| = r$ and the direction of the position vector changes from time to time.

(i) Displacement **:** The change of position vector or the displacement $\Delta\vec{r}$ of the particle from position *A* to the position *B* is given by referring the figure.

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 \implies \Delta r = |\Delta\vec{r}| = |\vec{r}_2 - \vec{r}_1|$$

$$\Delta r = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos\theta}$$

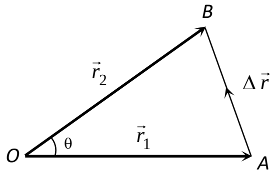
Putting $r_1 = r_2 = r$ we obtain

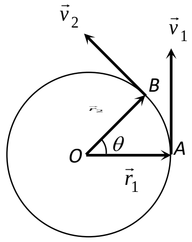
$\Delta r = \sqrt{r^2 + r^2 - 2r.r\cos\theta}$

$\Rightarrow \Delta r = \sqrt{2r^2(1-\cos\theta)}$

$= \sqrt{2r^2\left(2\sin^2\frac{\theta}{2}\right)}$

$\Delta r = 2r\sin\frac{\theta}{2}$





(ii) Distance **:** The distanced covered by the particle during the time t is given as

d = length of the arc $AB = r\ \theta$

(iii) Ratio of distance and displacement **:** $\frac{d}{\Delta r} = \frac{r\theta}{2r\sin\theta/2} \quad = \frac{\theta}{2}\text{cosec}\,(\theta/2)$

(2) **Angular displacement (θ) :** The angle turned by a body moving in a circle from some reference line is called angular displacement.

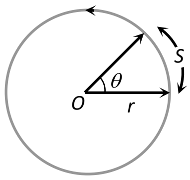
(i) Dimension = $[M^0L^0T^0]$ (as $\theta = arc/\text{radius})$.

(ii) Units = Radian or Degree. It is some time also specified in terms of fraction or multiple of revolution.

(iii) $2\pi\,\text{rad} = 360^o = 1\,\text{Revolution}$

(iv) Angular displacement is a axial vector quantity.

Its direction depends upon the sense of rotation of the object and can be given by Right Hand Rule; which states that if the curvature of the fingers of right hand represents the sense of rotation of the object, then the thumb, held perpendicular to the curvature of the fingers, represents the direction of angular displacement vector.



(v) Relation between linear displacement and angular displacement $\vec{s} = \vec{\theta}\times\vec{r}$

or $\qquad s = r\theta$

(3) **Angular velocity (ω) :** Angular velocity of an object in circular motion is defined as the time rate of change of its angular displacement.

(i) Angular velocity $\omega = \frac{\text{angle traced}}{\text{time taken}} = \underset{\Delta t\to 0}{Lt}\frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$

$\therefore\ \omega = \frac{d\theta}{dt}$

(ii) Dimension : $[M^0L^0T^{-1}]$

(iii) Units : Radians per second ($rad.s^{-1}$) or Degree per second.

(iv) Angular velocity is an axial vector.

Its direction is the same as that of $\Delta\theta$. For anticlockwise rotation of the point object on the circular path, the direction of ω, according to Right hand rule is along the axis of circular path directed upwards. For clockwise rotation of the point object on the circular path, the direction of ω is along the axis of circular path directed downwards.

(v) Relation between angular velocity and linear velocity $\vec{v} = \vec{\omega} \times \vec{r}$

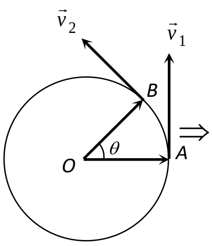
(vi) For uniform circular motion ω remains constant where as for non-uniform motion ω varies with respect to time.

❑It is important to note that nothing actually moves in the direction of the angular velocity vector $\vec{\omega}$. The direction of $\vec{\omega}$ simply represents that the circular motion is taking place in a plane perpendicular to it.

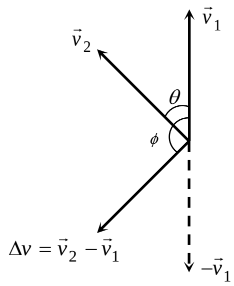
(4) **Change in velocity :** We want to know the magnitude and direction of the change in velocity of the particle which is performing uniform circular motion as it moves from A to B during time t as shown in figure. The change in velocity vector is given as

$$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$$

or



$$\Rightarrow \Delta v = \sqrt{v_1^2 + v_2^2 - 2v_1v_2\cos\theta}$$



For uniformm circular motion $v_1 = v_2 = v$

So $\Delta v = \sqrt{2v^2(1-\cos\theta)} = 2v\sin\frac{\theta}{2}$

The direction of $\Delta\vec{v}$ is shown in figure that can be given as

$$\phi = \frac{180^o - \theta}{2} = (90^o - \theta/2)$$

(5) **Time period (*T*) :** In circular motion, the time period is defined as the time taken by the object to complete one revolution on its circular path.

(i) Units : second.

(ii) Dimension : $[M^0L^0T]$

(iii) Time period of second's hand of watch = 60 *second*.

(iv) Time period of minute's hand of watch = 60 *minute*

(v) Time period of hour's hand of watch = 12 *hour*

(6) **Frequency (*n*) :** In circular motion, the frequency is defined as the number of revolutions completed by the object on its circular path in a unit time.

(i) Units : s^{-1} or hertz (Hz).

(ii) Dimension : $[M^0L^0T^{-1}]$

❑Relation between time period and frequency : If n is the frequency of revolution of an object in circular motion, then the object completes n revolutions in 1 second. Therefore, the object will complete one revolution in $1/n$ second.

$\therefore T = 1/n$

❑ Relation between angular velocity, frequency and time period : Consider a point object describing a uniform circular motion with frequency n and time period T. When the object completes one revolution, the angle traced at its axis of circular motion is 2π radians. It means, when time $t = T$, $\theta = 2\pi$ radians. Hence, angular velocity $\omega = \frac{\theta}{t} = \frac{2\pi}{T} = 2\pi n$ $(\because T = 1/n)$

$$\omega = \frac{2\pi}{T} = 2\pi n$$

❑ If two particles are moving on same circle or different coplanar concentric circles in same direction with different uniform angular speeds ω_A and ω_B respectively, the angular velocity of B relative to A will be

$$\omega_{\text{rel}} = \omega_B - \omega_A$$

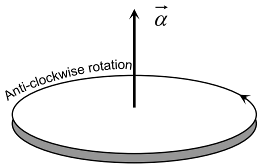
So the time taken by one to complete one revolution around O with respect to the other (i.e., time in which B complete one revolution around O with respect to the other (*i.e.*, time in which B completes one more or less revolution around O than A)

$$T = \frac{2\pi}{\omega_{\text{rel}}} = \frac{2\pi}{\omega_2 - \omega_1} = \frac{T_1 T_2}{T_1 - T_2} \qquad \left[\text{as } T = \frac{2\pi}{\omega}\right]$$

Special case : If $\omega_B = \omega_A, \omega_{\text{rel}} = 0$ and so $T = \infty$., particles will maintain their position relative to each other. This is what actually happens in case of geostationary satellite ($\omega_1 = \omega_2$ = constant)

(7) **Angular acceleration (α) :** Angular acceleration of an object in circular motion is defined as the time rate of change of its angular velocity.

(i) If $\Delta\omega$ be the change in angular velocity of the object in time interval Δt, while moving on a circular path, then angular acceleration of the object will be



$$\alpha = \underset{\Delta t \to 0}{Lt} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

(ii) Units : *rad. s*$^{-2}$

(iii) Dimension : $[M^0 L^0 T^{-2}]$

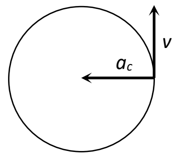
(iv) Relation between linear acceleration and angular acceleration $\vec{a} = \vec{\alpha} \times \vec{r}$

(v) For uniform circular motion since ω is constant so $\alpha = \frac{d\omega}{dt} = 0$

(vi) For non-uniform circular motion $\alpha \neq 0$

Centripetal Acceleration

(1) Acceleration acting on the object undergoing uniform circular motion is called centripetal acceleration.



(2) It always acts on the object along the radius towards the centre of the circular path.

(3) Magnitude of centripetal acceleration,

$$a = \frac{v^2}{r} = \omega^2 r = 4\pi^2 n^2 r = \frac{4\pi^2}{T^2} r$$

(4) Direction of centripetal acceleration : It is always the same as that of $\Delta\vec{\upsilon}$. When Δt decreases, $\Delta\theta$ also decreases. Due to which $\Delta\vec{\upsilon}$ becomes more and more perpendicular to $\vec{\upsilon}$. When $\Delta t \to 0$, $\Delta\vec{\upsilon}$ becomes perpendicular to the velocity vector. As the velocity vector of the particle at an instant acts along the tangent to the circular path, therefore $\Delta\vec{\upsilon}$ and hence the centripetal acceleration vector acts along the radius of the circular path at that point and is directed towards the centre of the circular path.

MULTIPUL CHOICE QUESTIONS

1. Position of a particle in a rectangular-co-ordinate system is (3, 2, 5). Then its position vector will be

(a) $3\hat{i}+5\hat{j}+2\hat{k}$ (b) $3\hat{i}+2\hat{j}+5\hat{k}$

(c) $5\hat{i}+3\hat{j}+2\hat{k}$ (d) None of these

2. If a particle moves from point P (2,3,5) to point Q (3,4,5). Its displacement vector be

(a) $\hat{i}+\hat{j}+10\hat{k}$ (b) $\hat{i}+\hat{j}+5\hat{k}$

(c) $\hat{i}+\hat{j}$ (d) $2\hat{i}+4\hat{j}+6\hat{k}$

3. The expression $\left(\frac{1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{2}}\hat{j}\right)$ is a

(a) Unit vector (b) Null vector

(c) Vector of magnitude $\sqrt{2}$ (d) Scalar

4. Any vector in an arbitrary direction can always be replaced by two (or three)

(a) Parallel vectors which have the original vector as their resultant
(b) Mutually perpendicular vectors which have the original vector as their resultant
(c) Arbitrary vectors which have the original vector as their resultant
(d) It is not possible to resolve a vector

5. The position vector of a particle is $\vec{r}=(a\cos\omega t)\hat{i}+(a\sin\omega t)\hat{j}$. The velocity of the particle is

(a) Parallel to the position vector
(b) Perpendicular to the position vector
(c) Directed towards the origin
(d) Directed away from the origin

6. If the resultant of the two forces has a magnitude smaller than the magnitude of larger force, the two forces must be

(a) Different both in magnitude and direction
(b) Mutually perpendicular to one another
(c) Possess extremely small magnitude
(d) Point in opposite directions

7. If $|\vec{A}+\vec{B}|=|\vec{A}|+|\vec{B}|$, then angle between $\vec{A}$ and $\vec{B}$ will be

(a) 90° (b) 120°

(c) 0° (d) 60°

8. The vector sum of two forces is perpendicular to their vector differences. In that case, the forces

(a) Are equal to each other in magnitude
(b) Are not equal to each other in magnitude
(c) Cannot be predicted
(d) Are equal to each other

9. If a vector $2\hat{i}+3\hat{j}+8\hat{k}$ is perpendicular to the vector $4\hat{j}-4\hat{i}+\alpha\hat{k}$. Then the value of α is

(a) –1 (b) $\frac{1}{2}$

(c) $-\frac{1}{2}$ (d) 1

10. If for two vectors $\vec{A}$ and $\vec{B}, \vec{A}\times\vec{B}=0,$ the vectors

(a) Are perpendicular to each other
(b) Are parallel to each other
(c) Act at an angle of 60°
(d) Act at an angle of 30°

11. What is the unit vector perpendicular to the following vectors $2\hat{i}+2\hat{j}-\hat{k}$ and $6\hat{i}-3\hat{j}+2\hat{k}$

(a) $\frac{\hat{i}+10\hat{j}-18\hat{k}}{5\sqrt{17}}$ (b) $\frac{\hat{i}-10\hat{j}+18\hat{k}}{5\sqrt{17}}$

(c) $\frac{\hat{i}-10\hat{j}-18\hat{k}}{5\sqrt{17}}$ (d) $\frac{\hat{i}+10\hat{j}+18\hat{k}}{5\sqrt{17}}$

12. Two cars are moving in the same direction with the same speed 30 *km/hr*. They are separated by a distance of 5 *km*, the speed of a car moving in the opposite direction if it meets these two cars at an interval of 4 minutes, will be

(a) 40 *km/hr* (b) 45 *km/hr*

(c) 30 *km/hr* (d) 15 *km/hr*

13. A man standing on a road hold his umbrella at 30° with the vertical to keep the rain away. He throws the umbrella and starts running at 10 *km/hr*. He finds that raindrops are hitting his head vertically, the speed of raindrops with respect to the road will be

(a) 10 *km/hr* (b) 20 *km/hr*

(c) 30 *km/hr* (d) 40 *km/hr*

14. In the above problem, the speed of raindrops *w.r.t.* the moving man, will be

(a) $10/\sqrt{2}\ km/h$ (b) 5 *km/h*

(c) $10\sqrt{3}\ km/h$ (d) $5/\sqrt{3}\ km/h$

15. A boat is moving with a velocity 3*i* + 4*j* with respect to ground. The water in the river is moving with a velocity – 3*i* – 4*j* with respect to ground. The relative velocity of the boat with respect to water is **[CPMT 1998]**

(a) 8*j* (b) – 6*i* – 8*j*

(c) 6*i* +8*j* (d) $5\sqrt{2}$

16. A 150 m long train is moving to north at a speed of 10 m/s. A parrot flying towards south with a speed of 5 m/s crosses the train. The time taken by the parrot the cross to train would be: **[**

(a) 30 s (b) 15 s

(c) 8 s (d) 10 s

17. A river is flowing from east to west at a speed of 5 m/min. A man on south bank of river, capable of swimming 10m/min in still water, wants to swim across the river in shortest time. He should swim

(a) Due north
(b) Due north-east

(c) Due north-east with double the speed of river

(d) None of these

18. A man can swim with velocity *v* relative to water. He has to cross a river of width *d* flowing with a velocity *u* (*u* > *v*). The distance through which he is carried down stream by the river is *x*. Which of the following statement is correct

(a) If he crosses the river in minimum time $x = \frac{du}{v}$

(b) *x* can not be less than $\frac{du}{v}$

(c) For *x* to be minimum he has to swim in a direction making an angle of $\frac{\pi}{2} + \sin^{-1}\left(\frac{v}{u}\right)$ with the direction of the flow of water

(d) x will be max. if he swims in a direction making an angle of $\frac{\pi}{2} + \sin^{-1}\frac{v}{u}$ with direction of the flow of water

19. A man sitting in a bus travelling in a direction from west to east with a speed of 40 *km*/*h* observes that the rain-drops are falling vertically down. To the another man standing on ground the rain will appear

(a) To fall vertically down

(b) To fall at an angle going from west to east

(c) To fall at an angle going from east to west

(d) The information given is insufficient to decide the direction of rain.

20. A stone of mass m is tied to a string of length l and rotated in a circle with a constant speed v. If the string is released, the stone flies

(a) Radially outward

(b) Radially inward

(c) Tangentially outward

(d) With an acceleration $\frac{mv^2}{l}$

21. A body is moving in a circular path with a constant speed. It has

(a) A constant velocity

(b) A constant acceleration

(c) An acceleration of constant magnitude

(d) An acceleration which varies with time

22. A motor cyclist going round in a circular track at constant speed has

(a)	Constant linear velocity(b)	Constant acceleration
(c)	Constant angular velocity	(d) Constant force

23. A particle P is moving in a circle of radius $'a'$ with a uniform speed v. C is the centre of the circle and AB is a diameter. When passing through *B* the angular velocity of P about A and C are in the ratio

(a)	1 : 1	(b)	1 : 2
(c)	2 : 1	(d)	4 : 1

24. A car moving on a horizontal road may be thrown out of the road in taking a turn

(a) By the gravitational force
(b) Due to lack of sufficient centripetal force
(c) Due to rolling frictional force between tyre and road
(d) Due to the reaction of the ground

25. Two particles of equal masses are revolving in circular paths of radii r_1 and r_2 respectively with the same speed. The ratio of their centripetal forces is

(a) $\frac{r_2}{r_1}$ (b) $\sqrt{\frac{r_2}{r_1}}$

(c) $\left(\frac{r_1}{r_2}\right)^2$ (d) $\left(\frac{r_2}{r_1}\right)^2$

26. A particle moves with constant angular velocity in a circle. During the motion its
(a) Energy is conserved
(b) Momentum is conserved
(c) Energy and momentum both are conserved
(d) None of the above is conserved

27. A stone tied to a string is rotated in a circle. If the string is cut, the stone flies away from the circle because
(a) A centrifugal force acts on the stone
(b) A centripetal force acts on the stone
(c) Of its inertia
(d) Reaction of the centripetal force

28. A body is revolving with a constant speed along a circle. If its direction of motion is reversed but the speed remains the same, then which of the following statement is true
(a) The centripetal force will not suffer any change in magnitude
(b) The centripetal force will have its direction reversed
(c) The centripetal force will not suffer any change in direction
(d) The centripetal force would be doubled

29. When a body moves with a constant speed along a circle
(a) No work is done on it
(b) No acceleration is produced in the body
(c) No force acts on the body
(d) Its velocity remains constant

30. A body of mass m moves in a circular path with uniform angular velocity. The motion of the body has constant
(a) Acceleration (b) Velocity
(c) Momentum (d) Kinetic energy

31. On a railway curve, the outside rail is laid higher than the inside one so that resultant force exerted on the wheels of the rail car by the tops of the rails will
(a) Have a horizontal inward component
(b) Be vertical
(c) Equilibriate the centripetal force
(d) Be decreased

32. If the overbridge is concave instead of being convex, the thrust on the road at the lowest position will be

(a) $mg + \frac{mv^2}{r}$ (b) $mg - \frac{mv^2}{r}$

(c) $\frac{m^2v^2g}{r}$ (d) $\frac{v^2g}{r}$

33. A cyclist taking turn bends inwards while a car passenger taking same turn is thrown outwards. The reason is

(a) Car is heavier than cycle
(b) Car has four wheels while cycle has only two
(c) Difference in the speed of the two
(d) Cyclist has to counteract the centrifugal force while in the case of car only the passenger is thrown by this force

34. A car sometimes overturns while taking a turn. When it overturns, it is

(a) The inner wheel which leaves the ground first
(b) The outer wheel which leaves the ground first
(c) Both the wheels leave the ground simultaneously
(d) Either wheel leaves the ground first

35. A particle is moving on a circular path with constant speed, then its acceleration will be

(a) Zero
(b) External radial acceleration
(c) Internal radial acceleration
(d) Constant acceleration

36. A train is moving towards north. At one place it turns towards north-east, here we observe that

(a) The radius of curvature of outer rail will be greater than that of the inner rail
(b) The radius of the inner rail will be greater than that of the outer rail
(c) The radius of curvature of one of the rails will be greater
(d) The radius of curvature of the outer and inner rails will be the same

37. The average acceleration vector for a particle having a uniform circular motion is

(a) A constant vector of magnitude $\frac{v^2}{r}$
(b) A vector of magnitude $\frac{v^2}{r}$ directed normal to the plane of the given uniform circular motion
(c) Equal to the instantaneous acceleration vector at the start of the motion
(d) A null vector

38. A vector is not changed if -

(1) It is rotated through an arbitrary angle
(2) It is multiplied by an arbitrary scale
(3) It is cross multiplied by a unit vector
(4) It is a slide parallel to itself

39. The component of a vector is -

(1) always less than its magnitude

(2) always greater than its magnitude

(3) always equal to its magnitude

(4) none of these

40. If $\vec{A} = \vec{B} + \vec{C}$ and the magnitudes $\vec{A}, \vec{B}$ and $\vec{C}$ are 5, 4 and 3 units, the angle between $\vec{A}$ and $\vec{C}$ is-

(1) $\cos^{-1}\left(\frac{3}{5}\right)$ (2) $\cos^{-1}\left(\frac{4}{5}\right)$

(3) $\frac{\pi}{2}$ (4) $\sin^{-1}\left(\frac{3}{4}\right)$

41. I started walking down a road to day-break facing the sun. After walking for some-time, I turned to my left, then I turned to the right once again. In which direction was I going then ?

(1) East (2) North-west (3) North-east (4) South

42. For a particle in circular motion the centripetal acceleration is

(a) Less than its tangential acceleration (b) Equal to its tangential acceleration

(c) More than its tangential acceleration (d) May be more or less than its tangential acceleration

43. Three particles A, B and C are projectied from the same point with the same initial speeds making angles 30°, 45° and 60° respectively with the horizontal. Which of the following statements is correct ?

(1) A, B and C have unequal ranges

(2) Ranges of A and C are equal and less than that of B

(3) Ranges of A and C are equal and greater than that of B

(4) A, B and C have equal ranges

44. A man travels 4 m due east and then turns by 90 degree and travels 3 due north the magnitude of displacement of the man is

a)1m b)7m c)5m (d)0

45. A can filled with water is revolved in a vertical circle of radius 4 metre and the water does not fall down. The time period of revolution will be

(a) 4 sec (b) 10 sec (c) 8 sec (d) 1 sec

46. Angle between equal vectors is

a)0 degree b) 30 degree c)90 degree d)180 degree

47. A force of 4N makes an angle 30 degree with x-axis. The y component of force is

a)$2\sqrt{3}$ N b)4N c)$2/\sqrt{3}$N d)2N

48. Which of the following is vector quantity?

a)Density b)Power c)Energy d)Momentum

49. A particle moves with constant speed but in constantly varying direction.The path of particle will be

a)elliptical b)linear c)circular d)parabolic

50. Time of flight of a projectile is 10 sec and its range is 500m.The maximum height reached by it is

a)50m b)80m c)100m d)125m

ASSERTION & REASONING QUESTIONS

These questions of two statements each, printed as Assertion and Reason. While answering these Questions you are required to choose any one of the following four responses.

(a)If both Assertion & Reason are true & the Reason is a correct explanation of the Assertion.

(b)If both Assertion and Reason are true but Reason is not a correct explanation of the Assertion.

(c)If Assertion is true but the Reason is false.

(d)If Assertion & Reason both are false

1. **Assertion :** If the initial and final positions coincide, the displacement is a null vector.
   Reason : A physical quantity can not be called a vector, if its magnitude is zero.
2. **Assertion :** A vector quantity is a quantity that has both magnitude and a direction and obeys the triangle law of addition or equivalently the parallelogram law of addition.

   Reason : The magnitude of the resultant vector of two given vectors can never be less than the magnitude of any of the given vector.
3. **Assertion :** The direction of a zero (null) vector is indeterminate.

   Reason : We can have $\vec{A}\times\vec{B}=\vec{A}.\vec{B}$ with A □ 0 and B □ 0.
4. **Assertion :** If the rectangular components of a force are 24 N and 7 N, then the magnitude of the force is 25 N.

   Reason : If $|\vec{A}|=|\vec{B}|$ = 1 then $|\vec{A}\times\vec{B}|^2$ + $|\vec{A}.\vec{B}|^2$ = 1.
5. **Assertion :** If three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ satisfy the relation $\vec{A}.\vec{B}$ **= 0** and $\vec{A}.\vec{C}$ **= 0** then the vector $\vec{A}$ may be parallel to $\vec{B}\times\vec{C}$.

   Reason : If $\vec{A}+\vec{B}=\vec{R}$ and **A + B = R**, then angle between $\vec{A}$ and $\vec{B}$ is zero.
6. **Assertion :** The angle between vectors $\vec{A}\times\vec{B}$ and $\vec{B}\times\vec{A}$ is □ radian.

   Reason : $\vec{B}\times\vec{A} = -\vec{A}\times\vec{B}$
7. **Assertion :** The minimum number of vectors of unequal magnitude required to produce zero resultant is three.
   Reason : Three vectors of unequal magnitude which can be represented by the three sides of a triangle taken in order, produce zero resultant.
8. **Assertion :** A vector can have zero magnitude if one of its components is not zero.
   Reason : Scalar product of two vectors cannot be a negative quantity.
9. **Assertion :** The angle between the two vectors $(\hat{i}+\hat{j})$ and $(\hat{j}+\hat{k})$ is $\frac{\pi}{3}$ radian.

Reason : Angle between two vectors $\vec{A}$ and $\vec{B}$ is given by □ = $\cos^{-1}\left(\frac{\vec{A}.\vec{B}}{AB}\right)$.

10. **Assertion :** Distance is a scalar quantity.
Reason : Distance is the length of path traversed.

11. **Assertion :** If position vector is given by $\vec{r} = \sin t\,\hat{i} + \cos t\,\hat{j} - 7t\,\hat{k}$, then magnitude of acceleration $|\vec{a}|$ =1.
Reason : The angles which the vector $\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$ makes with the coordinate axes are given by cos□ = $\frac{A_1}{A}$, cos □ = $\frac{A_2}{A}$ and cos □ = $\frac{A_3}{A}$.

12. **Assertion :** Adding a scalar to a vector of the same dimensions is a meaningful algebraic operation.
Reason : The displacement can be added with distance.

13. **Assertion :** Vector $(\hat{i} + \hat{j} + \hat{k})$ is perpendicular to $(\hat{i} - 2\hat{j} + \hat{k})$.
Reason : Two non-zero vectors are perpendicular if their dot product is equal to zero.

14. **Assertion :** The dot product of one vector with another vector may be a scalar or a vector.
Reason : If the product of two vectors is a vector quantity, then product is called a dot product.

15. Assertion : A physical quantity can be regarded as a vector, if magnitude as well as direction is associated with it.
Reason : A physical quantity can be regarded as a scalar quantity, if it is associated with magnitude only.

16. Assertion : In projectile motion, the angle between the instantaneous velocity and acceleration at the highest point is 180°.
Reason : At the highest point, velocity of projectile will be in horizontal direction only.

17. Assertion : Two particles of different mass, projected with same velocity at same angles. The maximum height attained by both the particle will be same.
Reason : The maximum height of projectile is independent of particle mass.

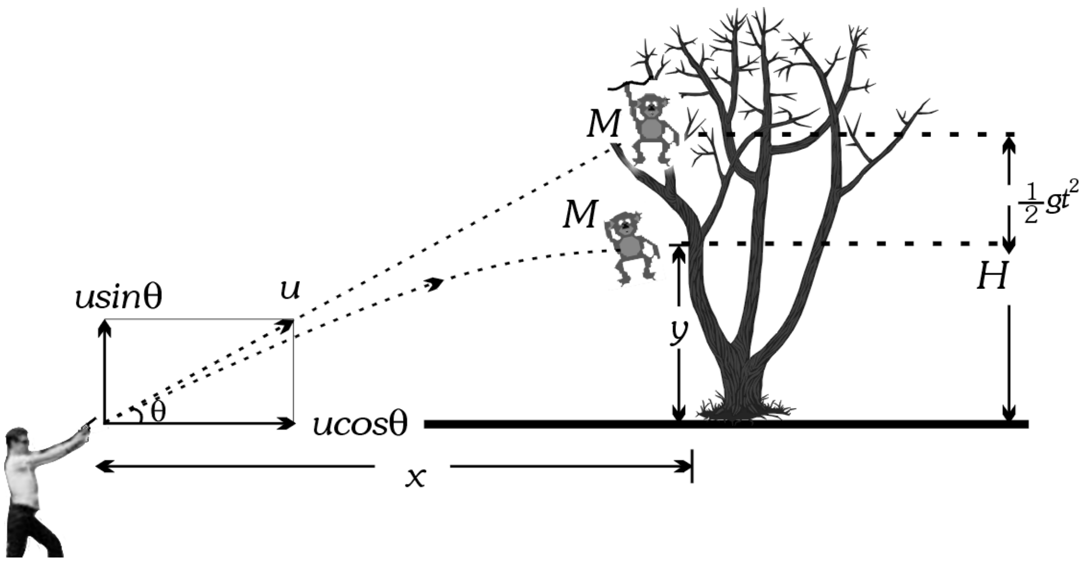
18. Assertion : The maximum horizontal range of projectile is proportional to square of velocity.
Reason : The maximum horizontal range of projectile is equal to maximum height attained by projectile.

19. Assertion : Horizontal range is same for angle of projection θ and $(90 - \theta)$.
Reason : Horizontal range is independent of angle of projection.

20. Assertion : For projection angle $\tan^{-1}(4)$, the horizontal range and the maximum height of a projectile are equal.
Reason :The maximum range of projectile is directly proportional to square of velocity and inversely proportional to acceleration due to gravity.

Case Study Based Questions

A hunter aims his gun and fires a bullet directly towards a monkey sitting on a distant tree. If the monkey remains in his position, he will be safe but at the instant the bullet leaves the barrel of gun, if the monkey drops from the tree, the bullet will hit the monkey because the bullet will not follow the linear path.



The path of motion of a bullet will be parabolic and this motion of bullet is defined as projectile motion.

If the force acting on a particle is oblique with initial velocity then the motion of particle is called projectile motion.Find the followings…

Q.1. Change in velocity (Between projection point and highest point)

(a) $\Delta\vec{u} = \vec{u}_f - \vec{u}_i = -u\sin\theta\,\hat{j}$ (b) $\Delta\vec{u} = \vec{u}_f - \vec{u}_i = u\sin\theta\,\hat{j}$

(c) $\Delta\vec{u} = u_f - u_i = -u\sin\theta\,\hat{i}$ (d) $\Delta\vec{u} = u_f - u_i = -2u\sin\theta\,\hat{i}$

Q.2. Change in velocity (Between complete projectile motion)

(a) $\Delta\vec{u} = u_f - u_i = -u\sin\theta\,\hat{i}$ (b) $\Delta\vec{u} = u_f - u_i = -2u\sin\theta\,\hat{i}$

(c) $\Delta\vec{u} = \vec{u}_f - \vec{u}_i = -u\sin\theta\,\hat{j}$ (d) $\Delta\vec{u} = \vec{u}_f - \vec{u}_i = u\sin\theta\,\hat{j}$

Q.3. Change in momentum (Between projection point and highest point)

(a) $\Delta\vec{p} = \vec{p}_f - \vec{p}_i = -mu\sin\theta\,\hat{j}$ (b) $\Delta\vec{p} = \vec{p}_f - \vec{p}_i = mu\sin\theta\,\hat{j}$

(c) $\Delta\vec{p} = \vec{p}_f - \vec{p}_i = -2mu\sin\theta\,\hat{j}$ (d) $\Delta\vec{p} = \vec{p}_f - \vec{p}_i = 2mu\sin\theta\,\hat{j}$

Q.4. Change in momentum (For the complete projectile motion)

(a) $\Delta\vec{p} = \vec{p}_f - \vec{p}_i = 2mu\sin\theta\,\hat{j}$ (b) $\Delta\vec{p} = \vec{p}_f - \vec{p}_i = -2mu\sin\theta\,\hat{j}$

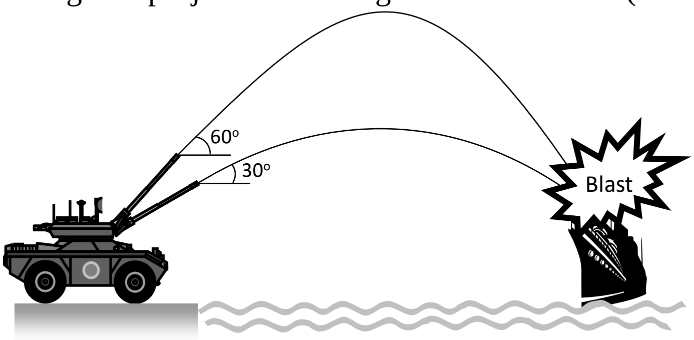
(c) $\Delta\vec{p} = \vec{p}_f - \vec{p}_i = -mu\sin\theta\,\hat{j}$ (d) $\Delta\vec{p} = \vec{p}_f - \vec{p}_i = mu\sin\theta\,\hat{j}$

Q.5. If a person can throw a projectile to a maximum distance R_{max}, The maximum height during the flight to which it will rise is

(a) $\left(\frac{R_{\max}}{4}\right)$ (b) $\left(\frac{R_{\max}}{2}\right)$

(c) $\left(\frac{R_{\max}}{5}\right)$ (d) $\left(\frac{R_{\max}}{11}\right)$

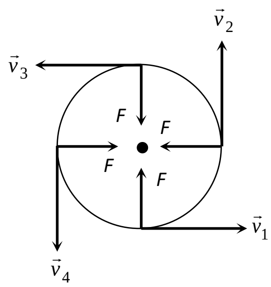
Q.6 If angle of projection is changed from θ to $\theta' = (90 - \theta)$ then range



(a)Remain Changed (b) Remain Unchanged

(c)Becomes two times (d) None

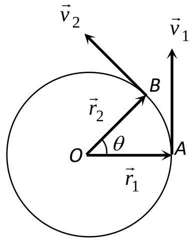
Circular motion is an example of motion in two dimensions. To create circular motion in a body it must be given some initial velocity and a force must then act on the body which is always directed at right angles to instantaneous velocity.



Since this force is always at right angles to the displacement therefore no work is done by the force on the particle. Hence, its kinetic energy and thus speed is unaffected. But due to simultaneous action of the force and the velocity the particle follows resultant path, which in this case is a circle.

Give the answer of followings …

Q.7 The change of position vector or the displacement $\Delta\vec{r}$ of the particle from position A to the position B is given by referring the figure



(a) $\Delta r = 2\sin\dfrac{\theta}{2}$ (b) $\Delta r = r\sin\dfrac{\theta}{2}$

(c) $\Delta r = 2r\sin\dfrac{\theta}{2}$ (d) $\Delta r = 2r\sin\dfrac{\theta}{4}$

Q.8.The magnitude of the change in velocity of the particle which is performing uniform circular motion as it moves from A to B during time t as shown in Q.7 figure. The change in velocity vector is given as

(a) $\Delta v = v\sin\dfrac{\theta}{2}$ (b) $\Delta v = -v\sin\dfrac{\theta}{2}$

(c) $\Delta v = 2v\sin\dfrac{\theta}{4}$ (d) $\Delta v = 2v\sin\dfrac{\theta}{2}$

Q.9. Acceleration acting on the object undergoing uniform circular motion always be along to

(a) along to tangent

(b) along the radius outwards the centre of the circular path.

(c) along the radius towards the centre of the circular path.

(d) None of these.

Q.10. Which of the following is/are wrong for circular motion

(a) For uniform circular motion since ω is constant so $\alpha = \dfrac{d\omega}{dt} = 0$

(b) Angular velocity is an axial vector.

(c) For uniform circular motion ω remains constant where as for non-uniform motion ω varies with respect to time.

(d) Angular displacement is a non pseudo vector quantity.

Scalar and Vector:-

In one dimensional motion of the objects only two directions are possible so the directional aspects of the quantities like displacement position velocity and acceleration can be described by using either positive or negative science physical quantity shown along positive direction will be given the positive sign whereas shown along negative direction possesses negative science but in case of motion of objects in two dimensions or in three dimension any object can have large number of directions so in order to deal with such situation we need to introduce the concept of new physical quantities in which we take care of both magnitude and direction in physics the physical quantities are broadly classified into categories scalars and vectors.

Magnitude of Resultant Vector and direction:-

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta} \qquad \tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

Q11.Angle between negative vectors is

a)0° b)60° c)90° d)180°

Q12.If resultant of two vectors of equal magnitude is equal to the magnitude of either vector then the angle between the two vectors is

a)30° b)90° c)60° d)180°

Q13.Which of the following is a scalar

a) displacement b) kinetic energy

c)couple d)momentum

Q14.Which of the following is not essential for three forces to produce zero resultant?

a) they should be in the same plane.

b) it should be possible to represent them by the three sides of a triangle taken in same order.

c) they should act along the sides of a parallelogram.

d) the resultant of any two forces should be equal and opposite to the third force.

Q15.What is the maximum number of rectangular components into which a vector can be resolved in a plane?

a) two b)three c)four d)Any number

Resolution of Vectors :-

A unit vector is a vector of unit magnitude and points in a particular direction.Unit vectors along the x, y and z axis of a rectangular co-ordinate system are denoted by $\hat{\imath}$, $\hat{\jmath}$,$\hat{k}$respectively. If a vector $\vec{A}$ subtends an angle α,β and γ with x, y and z axis respectively, then magnitude of its components along the three axes are A_x =A cosα ,A_y =A cosβ and A_z =A cos γ and the given vector $\vec{A}$ may be expressed as $\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$.

Process of vector addition and subtraction becomes very simple because now we can add or subtract the respective components of given vectors. You are now given two vectors $\vec{A} = 2\hat{\imath} + 3\,\hat{\jmath} + 4\hat{k}$

$\vec{B} = 32\hat{\imath} - 2\,\hat{\jmath} + \hat{k}$

Q16. The value of $\vec{A} + \vec{B}$ is

a) $5\hat{\imath} + \hat{\jmath} + 5\hat{k}$

b) $5\hat{\imath} - \hat{\jmath} + 5\hat{k}$

c) $\hat{\imath} + \hat{\jmath} + 5\hat{k}$

d)none of these

Q17. The magnitude of $|\vec{A} + \vec{B}|$ is

a)$\sqrt{50}$ b)$\sqrt{51}$ c)$\sqrt{49}$ d)$\sqrt{1}$

Q18. Value of $\vec{A} - \vec{B}$ is

a) $-\hat{\imath} + 5\,\hat{\jmath} + 3\hat{k}$
b) $\hat{\imath} + 5\,\hat{\jmath} + 3\hat{k}$
c) $-\hat{\imath} + \hat{\jmath} + 5\hat{k}$
d) None of these

Q19.The magnitude of $|\vec{A} - \vec{B}|$ is

a)$\sqrt{35}$ b}$\sqrt{17}$ c)$\sqrt{14}$ d)None of these

Q20. Vector addition

a)obeys commutative law b) does not obey commutative law

c) sometime obeys and sometime doesn't obey commutative law d) none of these

ANSWER KEY

Multiple Choice Questions

Q.No	1	2	3	4	5	6	7	8	9	10
Ans.	B	c	a	c	d	d	c	a	C	b
Q.No	11	12	13	14	15	16	17	18	19	20
Ans.	C	b	b	c	c	d	a	a	C	c
Q.No	21	22	23	24	25	26	27	28	29	30
Ans.	C	c	b	b	a	a	c	a	A	d
Q.No	31	32	33	34	35	36	37	38	39	40
Ans.	A	a	d	a	a	d	c	a	C	a
Q.No	41	42	43	44	45	46	47	48	49	50
Ans.	C	d	b	c	a	a	d	d	C	d

ASSERTION & REASONING QUESTIONS

Q.No	1	2	3	4	5	6	7	8	9	10
Ans.	3	3	3	2	2	1	2	4	1	1
Q.No	11	12	13	14	15	16	17	18	19	20
Ans.	2	4	1	4	2	4	1	3	1	3

Case Study Based Questions

Q.No	1	2	3	4	5	6	7	8	9	10
Ans.	a	B	a	b	a	b	c	d	C	d
Q.No	11	12	13	14	15	16	17	18	19	20
Ans.	d	d	b	c	a	a	b	a	A	b