KENDRIYA VIDYALAYA SANGHATHAN

CHANDIGARH REGION

CLASS-XI

SUBJECT-PHYSICS

Chapter -7

System of Particles and Rotational Motion

INDEX

1. GIST OF THE CHAPTER

SYSTEM OF PARTICLES AND ROTATIONAL MOTION

• A rigid body is a body with a perfectly definite and unchanging shape. The distances between all pairs of particles of such a body do not change.

  • Centre of Mass

For a system of particles, the centre of mass is defined as that point where the entire mass of the system is imagined to be concentrated, for consideration of its translational motion.
If all the external forces acting on the body/system of bodies were to be applied at the centre of mass, the state of rest/ motion of the body/system of bodies shall remain unaffected.

  • Centre of Mass of two particle system

• The centre of mass of a body or a system is its balancing point. The centre of mass of a two- particle system always lies on the line joining the two particles and is somewhere in between the particles.

If there are two particles of masses m_1 and m_2 having position vectors $\vec{r}_1$ and $\vec{r}_2$, then the position vector of the centre of mass is given by

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

Special Note: If the masses are of equal magnitude the centre of mass lies at the mid-point of the line joining them. If the masses are unequal, centre of mass is closer to the heavier body.

• For a system of n particles of masses $m_1, m_2, m_3 \ldots\ldots m_n$ and their respective position vectors $\vec{r}_1, \vec{r}_2, \vec{r}_3 \ldots\ldots \vec{r}_n$; the position

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots\ldots + m_n\vec{r}_n}{m_1 + m_2 + \ldots\ldots m_n} = \frac{\sum\limits_{i=1}^{n} m_i\vec{r}_i}{\sum\limits_{i=1}^{n} m_i}$$

• The co-ordinates of the centre of mass of an n-particle system is given as:

$$X = \frac{m_1 x_1 + m_2 x_2 + \ldots\ldots m_n x_n}{m_1 + m_2 + \ldots\ldots m_n} = \frac{\sum\limits_{i=1}^{n} m_i x_i}{\sum\limits_{i=1}^{n} m_i} = \frac{1}{M} \sum\limits_{i=1}^{n} m_i x_i$$

where $\sum\limits_{i=1}^{n} m_i = M$, mass of system.

$$Y = \frac{m_1 y_1 + m_2 y_2 + \ldots\ldots m_n y_n}{m_1 + m_2 + \ldots\ldots m_n} = \frac{\sum\limits_{i=1}^{n} m_i y_i}{\sum\limits_{i=1}^{n} m_i} = \frac{1}{M} \sum\limits_{i=1}^{n} m_i y_i$$

$$Z = \frac{m_1 z_1 + m_2 z_2 + \ldots\ldots m_n z_n}{m_1 + m_2 + \ldots\ldots m_n} = \frac{\sum\limits_{i=1}^{n} m_i z_i}{\sum\limits_{i=1}^{n} m_i} = \frac{1}{M} \sum\limits_{i=1}^{n} m_i z_i$$

• Momentum Conservation and Motion of Centre of Mass

The centre of mass of a system of particles moves as if the entire mass of the system were concentrated at the centre of mass and all the external forces were applied at that point. Velocity of centre of mass of a system of two particles, m_1 and m_2 with velocity v_1 and v_2 is given by,

$$V_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$$

Acceleration of centre of mass, a_{cm} of a two body system is given by

$$a_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2}$$

• If no external force acts on the body, then the centre of mass will have constant momentum. Its velocity is constant and acceleration is zero, i.e., MV_{cm} = constant.

- **Centre of mass of a rigid body**

   Rigid Body: An extended body is also a system of an infinitely large number of particles having an infinitely small separation between them. When a body deforms, the separation between the distance between its particles and their relative locations changes. A rigid body is an extended object in which separations and relative location of all of its constituent particles remain the same under all circumstances.
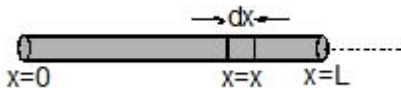
- **Centre of Mass of a uniform rod**

Suppose a rod of mass M and length L is lying along the x-axis with its one end at x = 0 and the other at x = L.
Mass per unit length of the rod l = M/L
Hence, dm, (the mass of the element dx situated at x = x is) = l dx
The coordinates of the element dx are (x, 0, 0). Therefore, x-coordinate of COM of the rod will be

$$X_{COM} = \frac{\int_0^L x\,dm}{\int_0^L dm} = \frac{\int_0^L (x)(\lambda\,dx)}{\int_0^L \lambda\,dx} = \frac{1}{L}\int_0^L x\,dx = \frac{L}{2}$$

The y-coordinate of COM is

$$Y_{COM} = \frac{\int y\,dm}{\int dm} = 0$$

Similarly, $z_{COM} = 0$

i.e., the coordinates of COM of the rod are (0, 0), i.e., it lies at the centre of the rod.

• Vector Product or Cross Product of two vectors

The vector product or cross product of two vectors $\vec{A}$ and $\vec{B}$ is another vector $\vec{C}$, whose magnitude is equal to the product of the magnitudes of the two vectors and sine of the smaller angle between them.

If θ is the smaller angle between $\vec{A}$ and $\vec{B}$, then

$$\vec{A} \times \vec{B} = \vec{C} = AB \sin \theta \, \hat{C}$$

where $\hat{C}$ is a unit vector in the direction of $\vec{C}$. The direction of $\vec{C}$ or $\hat{C}$ (i.e., vector product of two vectors) is perpendicular to the plane containing $\vec{A}$ and $\vec{B}$ and pointing in the direction of advance of a right handed screw when rotated from $\vec{A}$ to $\vec{B}$.

● Some important properties of cross-product are as follows:

(a) For parallel as well as anti-parallel vectors (i.e., when θ = 0° or 180°), the cross-product is zero.

(b) The magnitude of cross-product of two perpendicular vectors is equal to the product of the magnitudes of the given vectors.

(c) Vector product is anti-commutative i.e., $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

(d) Vector product is distributive i.e., $\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$

(e) $\vec{A} \times \vec{B}$ does not change sign under reflection i.e., $(-\vec{A}) \times (-\vec{B}) = \vec{A} \times \vec{B}$

(f) For unit orthogonal vectors, we have

$$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0, \quad \hat{i} \times \hat{j} = \hat{k}, \quad \hat{j} \times \hat{k} = \hat{i} \quad \text{and} \quad \hat{k} \times \hat{i} = \hat{j}$$

Moreover $\quad \hat{j} \times \hat{i} = -\hat{k}, \quad \hat{k} \times \hat{j} = -\hat{i} \quad \text{and} \quad \hat{i} \times \hat{k} = -\hat{j}$

(g) In terms of components $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$.

● The angular velocity of a body or a particle is defined as the ratio of the angular displacement of the body or the particle to the time interval during which this displacement occurs.

$$\omega = \frac{d\theta}{dt}$$

The direction of angular velocity is along the axis of rotation. It is measured in radian/sec and its dimensional formula is $[M^0 L^0 T^{-1}]$.

The relation between angular velocity and linear velocity is given by

$$\boxed{\vec{v} = \vec{\omega} \times \vec{r}}$$

● The angular acceleration of a body is defined as the ratio of the change in the angular velocity to the time interval.

$$\text{Angular acceleration} = \frac{\text{Change in angular velocity}}{\text{time taken}}$$

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt}$$

The unit of angular acceleration is rad s^{-2} and dimensional formula is $[M^0 L^0 L^{-2}]$.

• Moment of force

The Moment of a force is a measure of its tendency to cause a body to rotate about a specific point or axis. ... The magnitude of the moment of a force acting about a point or axis is directly protinoal to the distance of the force from the point or axis.
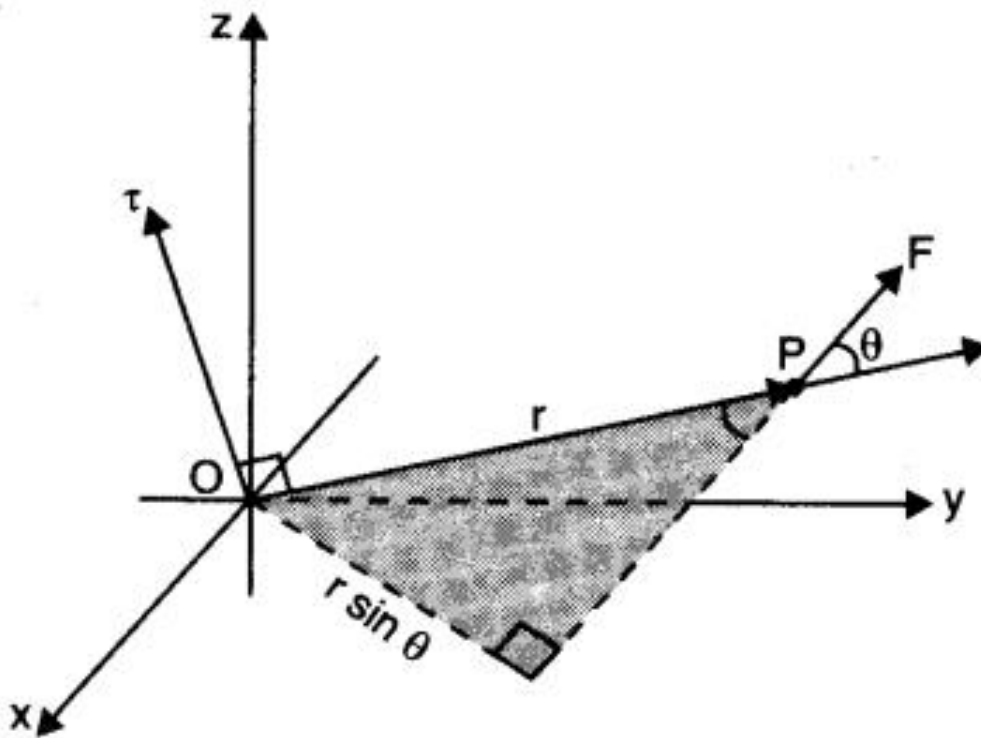
• Torque

Torque is the moment of force. Torque acting on a particle is defined as the product of the magnitude of the force acting on the particle and the perpendicular distance of the application of force from the axis of rotation of the particle.

Torque or moment of force = force × perpendicular distance

$$\vec{\tau} = \vec{r} \times \vec{F} = rF \sin \theta \, \hat{n}$$

where θ is smaller angle between $\vec{r}$ and $\vec{F}$; $\hat{n}$ is unit vector along $\vec{r}$.

It is measured in Nm and has dimensions of $[ML^2T^{-2}]$.

• Angular Momentum and law of conservation of angular momentum

The angular momentum (or moment of momentum) about an axis of rotation is a vector quantity, whose magnitude is equal to the product of the magnitude of momentum and the perpendicular distance of the line of action of momentum from the axis of rotation and its direction is perpendicular to the plane containing the momentum and the perpendicular distance.

It is given by

$$\vec{L} = \vec{r} \times \vec{p}$$

SI unit of angular momentum is kg m^{2}s^{-1} and its dimensional formula is [M^{1}L^{2}T^{-1}].
- Geometrically, the angular momentum of a particle is equal to twice the product of its mass and the areal velocity, i.e.,

$$L = 2 \, m \times \frac{dA}{dt}$$

- Torque (τ) and angular momentum are corelated as:

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

- If no net external torque acts on a system then the total angular momentum of the system remains conserved. Mathematically, if $\vec{\tau}_{ext} = \vec{0}$, then

$$\vec{L} = \vec{L}_1 + \vec{L}_2 + \text{........} + \vec{L}_n = \text{a constant}$$

If the moment of inertia of the body changes from I_1 to I_2 due to the change of the distribution of mass of the body, then angular velocity of the body changes from $\vec{\omega}_1$ to $\vec{\omega}_2$, such that

$$I_1 \, \vec{\omega}_1 = I_2 \vec{\omega}_1 \quad \text{or} \quad I_1 \omega_1 = I_2 \omega_2.$$

- **Equilibrium of rigid bodies**

A rigid body is in equilibrium when it is not undergoing a change in rotational or translational motion. This equilibrium requires that two conditions must be met.
The first condition is related to the translational motion. The vector sum of the forces on the body must be zero:

$$\sum F = 0$$

The second condition is related to the rotational motion. When the forces do not act through a common point or pivot, they may cause the body to rotate, even though the vector sum of the forces may be zero.
. A net torque will cause a body, initially at rest, to undergo rotation.
The second condition for static equilibrium is: The sum of the all the torques (due to each of the forces on the body) must be zero:

$$\sum \tau = 0$$

• **Axis of Rotation**

A rigid body is said to be rotating if every point mass that makes it up, describes a circular path of a different radius but the same angular speed. The circular paths of all the point masses have a common centre. A line passing through this common centre is the axis of rotation.
• A rigid body is said to be in equilibrium if under the action of forces/torques, the body remains in its position of rest or of uniform motion.
For translational equilibrium, the vector sum of all the forces acting on a body must be zero. For rotational equilibrium, the vector sum of torques of all the forces acting on that body about the reference point must be zero. For complete equilibrium, both these conditions must be fulfilled.

• **Moment of Inertia**
The rotational inertia of a rigid body is referred to as its moment of inertia.
The moment of inertia of a body about an axis is defined as the sum of the products of the masses of the particles constituting the body and the square of their respective perpendicular distance from the axis.
It is given by .

$$I = m_1 r_1^2 + m_2 r_2^2 + \cdots\cdots + m_n r_n^2 = \sum_{i=1}^{n} m_i r_i^2,$$

where m_i is the mass and r_i the distance of the ith particle of the rigid body from the axis of rotation.
It is measured in kg m^2 and has the dimension of $[ML^2]$.

• Radius of Gyration

The distance of a point in a body from the axis of rotation, at which if whole of the mass of the body were supposed to be concentrated, its moment of inertia about the axis of rotation would be the same as that determined by the actual distribution of mass of the body is called radius of gyration.
If we consider that the whole mass of the body is concentrated at a distance K from the axis of rotation, then moment of inertia I can be expressed as $I = MK^2$
where M is the total mass of the body and K is the radius of gyration. It is given as

$$K = \sqrt{\left(\frac{r_1^2 + r_2^2 + \cdots. + r_n^2}{n} \right)}$$

• Theorem of Parallel Axes

According to this theorem, the moment of inertia I of a body about any axis is equal to its moment of inertia about a parallel axis through centre of mass, I_{cm}, plus Ma^2 where M is the mass of the body and V is the perpendicular distance between the axes, i.e.,
$I = I_{cm} + Ma^2$

• Theorem of Perpendicular Axes

According to this theorem, the moment of inertia I of the body about a perpendicular axis is equal to the sum of moments of inertia of the body about two axes at right angles to each other in the plane of the body and intersecting at a point where the perpendicular axis passes, i.e.,

$$I = I_x + I_y$$

● A body in rotatory motion possesses rotational kinetic energy given by:

$$\text{Rotational } K.E. = \frac{1}{2} I \omega^2.$$

● In terms of moment of inertia of a body, its angular momentum is defined as the product of moment of inertia and angular velocity *i.e.,*

$$\vec{L} = I \vec{\omega}$$

● Torque may be defined as the produce of moment of inertia and the angular acceleration *i.e.,*

$$\vec{\tau} = I \vec{\alpha}$$

2. Formulae Used in Chapter

Values of moments of inertia for simple geometrical objects .

S.NO.	OBJECT	AXIS	Moment of Inertia
1.	Uniform rod of length l	perpendicular to rod through its centre	$\frac{1}{12}Ml^2$
2.	Uniform rectangular lamina of length l and breadth b	perpendicular to lamina and through its centre	$M\left(\frac{l^2+b^2}{12}\right)$
3.	Uniform circular ring of radius R	perpendicular to its plane and through the centre	MR^2
4.	Uniform circular ring of radius R	Diameter	$MR^2/2$
5.	Uniform circular disc of radius R	perpendicular to its plane and through the centre	$\frac{1}{2}MR^2$
6.	Uniform circular disc of radius R	Diameter	$\frac{1}{4}MR^2$
7.	Hollow cylinder of radius R	Axis of cylinder	MR^2
8.	Solid cylinder of radius R	Axis of cylinder	$\frac{1}{2}MR^2$
9.	Hollow sphere of radius R	Diameter	$\frac{2}{3}MR^2$
10.	Solid sphere of radius R	Diameter	$\frac{2}{5}MR^2$

• IMPORTANT TABLES

TABLE 7.1

Quantity	Symbol	Dimensions	Units	Remarks
Angular velocity	ω	$[T^{-1}]$	rad s^{-1}	$\vec{v} = \vec{\omega} \times \vec{\gamma}$
Angular Momentum	L	$[ML^2T^{-1}]$	J s	$\vec{L} = \vec{r} \times \vec{p}$
Torque	τ	$[ML^2T^{-2}]$	N m	$\vec{\tau} = \vec{r} \times \vec{F}$
Moment of inertia	I	$[ML^2]$	kg m^2	$I = \sum m_i r_i^2$

TABLE 7.2 Analogy between linear motion and rotational motion

	Linear Motion		Rotational Motion
1.	Distance/displacement (s)	1.	Angle or angular displacement (θ)
2.	Linear velocity $v = \dfrac{ds}{dt}$	2.	Angular velocity $\omega = \dfrac{d\theta}{dt}$
3.	Linear acceleration $a = \dfrac{dv}{dt} = \dfrac{d^2s}{dt^2}$	3.	Angular acceleration $\alpha = \dfrac{d\omega}{dt} = \dfrac{d^2\theta}{dt^2}$
4.	Mass (m)	4.	Moment of inertia (I)
5.	Linear momentum $P = mv$	5.	Angular momentum $L = I\omega$
6.	Force $F = ma$	6.	Torque $\tau = I\,\alpha$
7.	Also, force $F = \dfrac{dp}{dt}$	7.	Also, torque $\tau = \dfrac{dL}{dt}$
8.	Translational K.E $= \dfrac{1}{2}mv^2 = \dfrac{p^2}{2m}$	8.	Rotational K.E $= \dfrac{1}{2}I\omega^2 = \dfrac{L^2}{2I}$
9.	Work done, $W = Fs$	9.	Work done, $W = \tau\,\theta$
10.	Power $P = Fv$	10.	Power $= \tau\,\omega$
11.	Linear momentum of a system is conserved when no external force acts on the system. (Principle of conservation of linear momentum)	11.	Angular momentum of a system is conserved when no external torque acts on the system. (Principle of conservation of angular momentum)
12.	Equations of Translational motion (i) $v = u + at$ (ii) $s = ut + \dfrac{1}{2}at^2$ (iii) $v^2 - u^2 = 2as$, where the symbols have their usual meaning.	12.	Equations of Rotational motion (i) $\omega_2 = \omega_1 + \alpha t$ (ii) $\theta = \omega_1 t + \dfrac{1}{2}\alpha t^2$ (iii) $\omega_2^2 - \omega_1^2 = 2\,\alpha\,\theta$, where the symbols have their usual meaning.
13.	Distance travelled in nth second $S_{nth} = u + \dfrac{a}{2}(2n-1)$	13.	Angle traced in nth second $\theta_{nth} = \omega_1 + \dfrac{\alpha}{2}(2n-1)$

3. MULTIPLE CHOICE QUESTIONS

1. What is the radius of gyration of a uniform rod whose length is L and passes through the centre of mass
(a) L/√3   (b) L/√2   (c) L/2√3   (d) $L^2/12$

2. Two identical particles move towards each other with velocity 3v and 2v respectively. The velocity of their centre of mass is
(a) 2.5v   (b) 3v   (c) 5v   (d) 1.5v

3. What is the angular momentum vector in an orbital motion?
(a) The vector is perpendicular to the orbital plane
(b) The vector is along the radius vector
(c) The vector is parallel to the linear momentum
(d) The vector is in the orbital plane

4. What is the acceleration of the rolling sphere at the centre of the plane with inclination, Ө to the horizontal?
(a) Zero   (b) Less than g sin Ө
(c) Greater than g sin Ө   (d) g sin Ө

5. On which of the following factor does the moment of inertia of an object not depend upon
(a) Axis of rotation   (b) Angular velocity
(c) Distribution of mass   (d) Mass of an object

6. What is the effect of frictional force on a round object with mass M and radius R which rolls down the inclined plane without slipping along the way
(a) There is a decrease in the rotational motion
(b) There is a decrease in the rotational and translational motion
(c) There is a conversion of translational motion into rotational motion
(d) Kinetic energy is converted into heat

7. When the torque acting on the system is zero, which of the following is constant?
(a) Linear impulse   (b) Linear momentum
(c) Force   (d) Angular momentum

8. The angular momentum of a rigid body is L and its kinetic energy is halved. What happens to its angular momentum?
(a) L   (b) 2L
(c) L/2   (d) L/4

9. Consider two objects a disk and a sphere that has the same radius but different masses which roll down the two inclined planes with the same altitude and length. Out of the two objects, which one gets to the bottom of the plane first?
(a) It is dependent on the masses of the objects
(b) Disk
(c) Sphere
(d) Both reach at the same time

10. What does $L^2/2I$ represent?
(a) Power   (b) The torque of a particle
(c) The potential energy of a particle (d) The rotational kinetic energy of a particle

11. The motion of planets in the solar system is an example of conservation of

(a) Energy   (b) Linear momentum
(c) Angular momentum   (d) Mass

12. When does the moment of inertia of a body come into the picture?
(a) When the motion is rotational   (b) When the motion is linear
(c) When the motion is along a curved path
(d) None of the above

13. A body of M.I. 3 kg m² rotating with an angular velocity 2 rad/s has the same K.E. as a mass of 12 kg moving with a velocity of
(a) 1 m/s   (b) 2 m/s
(c) 4 m/s   (d) 8 m/s

14. A particle performing uniform circular motion has angular momentum L. If its angular frequency is doubled and its kinetic energy halved, then the new angular momentum is
(a) L/2   (b) L/4
(c) 2 L   (d) 4 L

15. A thin uniform, circular ring is rolling down an inclined plane of inclination 30^0 without slipping. Its linear acceleration along the inclined plane will be
(a) g/2   (b) g/3
(c) g/4   (d) 2g/3

16. A ring of radius r and mass m rotates about an axis passing through its centre and perpendicular to its plane with angular velocity ω. Its kinetic energy is
(a) $mr\omega^2$ (b) $\frac{1}{2}mr\omega^2$

(c) $I\omega^2$ (d) $\frac{1}{2}I\omega^2$

17.The moment of inertia of a disc having mass M and radius R, about an axis passing through its centre and perpendicular to its plane is
(a) $\frac{1}{2}MR^2$ (b) MR^2

(c) $\frac{1}{2}MR^2$ (d) $\frac{5}{4}MR^2$

18. If a body is rotating about an axis, passing through its centre of mass then its angular momentum is directed along its
(a) Radius   (b) Tangent
(c) Circumference   (d) Axis of rotation

19. A solid cylinder of mass 20 kg, has length 1 metre and radius 0.5m. then its momentum of inertia in kg m² about its geometrical axis is
(a) 2.5   (b) 5
(c) 1.5   (d) 3

20. A particle moves on a circular path with decreasing speed. Choose the correct statement.
(a) Angular momentum remains constant.
(b) Acceleration is towards the centre.
(c) Particle moves on a spiral path with decreasing radius.
(d) The direction of angular momentum remains constant.

21. Rotational analogue of mass in linear motion is
(a) Weight   (b) Moment of inertia
(c) Torque   (d) Angular momentum

22. Which is the wrong relation from the following?
(a) $\tau = I\,a$ (b) F = ma
(c) L = I w   (d) I = $\tau\,a$

23. A solid sphere is rotating in free space. If the radius of the sphere is increased keeping mass same, which one of the following will not be affected?
(a) Moment of inertia   (b) Angular momentum
(c) Angular velocity   (d) Rotational kinetic energy

24. The moment of momentum is also called as
(a) Couple   (b) torque
(c) impulse   (d) angular momentum

25. Larger the moment arm, the greater will be the
(a) Momentum   (b) Velocity
(c) Torque   (d) Axis of rotation

26. The angular speed of minute arm in a watch is :
(a) $\pi/21600$ rad s^{-1} (b) $\pi/12$ rad s^{-1}
(c) $\pi/3600$ rad s^{-1} (d) $\pi/1800$ rad s^{-1}

27. One circular ring and one circular disc both having same mass and radius. What will be the ratio of their moment of inertia about the axis passing through their centres and perpendicular to their planes?

(a) 1:1   (b) 2:1
(c) 1:2   (d) 1:$\sqrt{2}$

28. If I, a and t are the moment of inertia, angular acceleration and torque respectively of a body rotating about any axis with angular velocity w, then
(a) t = Ia   (b) t = Iw
(c) I = tw   (d) a = Iw

29. A dancer on ice spins faster when she folds here arms. This is due to
(a) Increases in energy and increase in angular momentum
(b) Decrease in friction at the skates
(c) Constant angular momentum and increase in kinetic energy
(d) Increase in energy an decreases in angular momentum

30. The moment of inertia of a uniform semicircular disc of mass M and radius about a line perpendicular to the plane of the disc through the centre is
(a) (2/5) Mr²   (b) (1/4) Mr power 2
(c) (1/2) Mr²   (d) Mr²

31. Choose the correct relation

(a) $\hat{i} \times \hat{j} = 0$

(b) $\hat{i} \times \hat{j} = -\hat{k}$

(c) $\hat{i} \times \hat{j} = \hat{k}$

(d) $\hat{i} \times \hat{j} = \hat{j}$

32. For increasing the angular velocity of an object by 10%, the kinetic energy has to be increased by
(a) 40%
(b) 20%
(c) 10%
(d) 21%

33. What does a couple produce?

a) pure rotation

b) rotation and translation

c) no motion

d) pure translation

34. Maintaining constant velocity, a particle is travelling down a line which is parallel to X-axis (positive). What will be the angular momentum magnitude with regards to origin?

a) zero

b) increasing along with x

c) decreasing along with x

d) remains constant

35. A rope coils an empty cylinder whose mass and radius are 3 kg and 40 cm respectively. Determine the cylinder's angular acceleration if someone pulls the rope with 30 N force.

a) 0.25 rad s^{-2}

b) 25 rad s^{-2}

c) 5 m s^{-2}

d) 25 m s^{-2}

36. A closed vessel of cylindrical shape is partly filled with water. What happens to the moment of inertia if the vessel rotates along a horizontal plane around perpendicular bisector?

a) increases

b) decreases

c) stays constant

d) depends on rotation direction

37. One rigid body having angular momentum L is rotating, and its K.E (kinetic energy) is divided into two parts. What will be its angular momentum then?

a) L

b) L^2

c) 2 L

d) L / 2

38. A particle is in UCM (uniform circular motion). Its angular momentum remains intact about:

a) middle point of circle

b) at any point within the circle

c) circumference point of the circle

d) at any point exterior to the circle.

39. An object round in shape having radius R and mass M rolls down by an inclined surface but does not slip. Its frictional force:

a) vanishes K.E as heat

b) reduces rotational motion

c) changes translational energy to rotational energy

d) reduces both translational and rotational motion.

40. What will be a body's angular momentum if the time period is doubled and its moment of inertia is kept constant?

a) remain constant

b) become half

c) doubles

d) quadruples

41. Centre of mass of two particle (different masses) system lies near to

a) heavier mass

b) lighter mass

c) at the middle only

d) always outside the body

42. Find the moment of inertia if 2000Nm torque is acting on a body with 2 rad/ s^2 angular acceleration.

a) 1200 kgm^2

b) 900 kgm^2

c) 1000 kgm^2

d) Cannot be determined

43. If an object's angular velocity is increased by 10 per cent, then K.E must be increased by what percentage?

a) 40 %

b) 20 %

c) 10 %

d) 21 %

44. What is angular momentum?

a) a scalar

b) scalar as well as vector

c) a polar vector

d) an axial vector

45. Choose the wrong relation to the following options.

a) a = rα

b) F = m x a

c) K.E. = $L^2/$ ω

d) $\tau = rXF$

46. Two discs A and B circular in shape have a uniform thickness and similar masses. But the density of disc 1 is greater than disc 2. What is the moment of inertia?

a) $I_1 > I_2$

b) $I_1 >> I_2$

c) $I_1 < I_2$

d) $I_1 = I_2$

47. Calculate the angular momentum of a body when it's K.E and M.I are 4 joules and 2 kg m^2 respectively.

a) 4 kg m^2 / sec

b) 5 kg m^2 / sec

c) 6 kg m^2 / sec

d) 7 kg m^2 / sec

48. Raw egg and hard boiled eggs are given equal torques,

a) Raw egg will be rotated faster

b) Hard boiled egg will be rotated faster.

c) angular velocity will be same for both.

d) Can't say anything.

49. A disc having diameter 2 m and mass 2 kg undergoes rotational motion. Consider the rotation of disc is from 300 to 600 rpm. Evaluate the work done.

a) 1479 J

b) 14.79 J

c) 147.9 J   d) 1.479 J

50. Suppose a gymnast is sitting on a rotating chair and his arms are outstretched. If he suddenly shortens his arms, what will happen?

a) angular velocity will decrease   b) moment of inertia will decrease

c) angular velocity will stay constant d) angular momentum will increase.

4. ASSERTION AND REASONING QUESTIONS

For question numbers 1 to 20, two statements are given-one labelled Assertion and the other labelled Reason. Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below:

    a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion
    b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion
    c) Assertion is true but Reason is false
    d) Assertion is false and Reason is also false

1. Assertion: Centre of mass of a system does not move under the action of internal forces.
Reason: Internal forces are non-conservative forces.

2. Assertion: For a system of particles under central force field, the total angular momentum is conserved.
Reason : The torque acting on such a system is zero.

3. Assertion: A satellite is orbiting about a planet then its angular momentum is conserved.

Reason: Linear momentum conservation leads to angular momentum conservation.

4. Assertion: If there is no external torque on a body about its centre of mass, then the velocity of the centre of mass remains constant.

Reason: Linear momentum of an isolated system remains constant.

5. Assertion: Two cylinders, one hollow (metal) and the other solid(wood) with the same mass and identical dimensions are simultaneously allowed to roll without slipping down an inclined plane form the same height. The hollow cylinder will reach the bottom of the inclined plane first.

Reason: By the principle of conservation of energy, the total kinetic energies of both the cylinders are identical when they reach the bottom of the incline.

6. Assertion: The angular velocity of a rigid body in motion is defined for the whole body.

Reason: All points on a rigid body performing pure rotational motion are having same angular velocity.

7. Assertion: In rolling, all points of a rigid body have same linear velocity.

Reason: The rotational motion does not affect the linear velocity.

8. Assertion: In rotational plus translational motion of a rigid body, different particles of the rigid body may have different velocities but they will have same accelerations.

Reason: Translational motion of a particle is equivalent to the translation motion of a rigid body.

9. Assertion: A body may be accelerated even when it is moving uniformly.
Reason: When direction of motion of the body is changing then body may have acceleration

10. Assertion: The size and the shape of the rigid body remains unaffected under the effect of external forces.
Reason: The distance between two particles remains constant in a rigid body

11. Assertion: There are very small sporadic changes in the period of rotation of the earth.
Reason: Shifting of large masses in the earth's atmosphere produces a change in the moment of inertia of the earth causing its period of rotation to change.

12. Assertion (A): A person standing on a rotating platform suddenly stretched his arms, the platform slows down.
Reason (R): A person by stretching his arms increases the moment of inertia and decreases angular velocity.

13. Assertion (A): If a particle moves with a constant velocity, then angular momentum of this particle about any point remains constant.
Reason (R): Angular momentum has the units of Planck's constant

14. Assertion (A): The speed of whirlwind in a tornado is alarmingly high.
Reason (R): If no external torque acts on a body, its angular velocity remains conserved.

15. Assertion (A): Power associated with torque is product of torque and angular speed of the body about the axis of rotation.
Reason (R): Torque in rotational motion is analogue to force in translatory motion.

16. Assertion (A): There are very small sporadic changes in the speed of rotation of the earth.
Reason (R): Shifting of large air masses in the earth's atmosphere produce a change in the moment of inertia of the earth causing its speed of rotation to change.

17.

Assertion (A) A uniform disc of radius R is performing impure rolling motion on a rough horizontal plane as shown in figure. After sometime the disc comes to rest. It is possible only when $v_0 = \dfrac{\omega_0 R}{2}$.
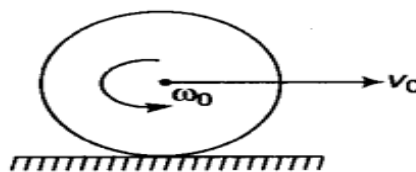
Reason (R) For a body performing pure rolling motion, the angular momentum is conserved about any point in space.

18. Assertion: To unscrew a rusted nut, we need a pipe wrench of longer arm.

Reason: Wrench with longer arm reduces the force applied on arm.

19. Assertion: A couple does not exert a net force even though it exerts a torque.

Reason: Couple is a pair of two forces with equal magnitude but opposite directions acting simultaneously on a body in different lines of action.

20. Assertion: A ladder is more apt to slip when you are high up on it than when you just begin to climb.

Reason: At the highest point on the ladder, the torque is large and on climbing up thetorque is small.

5. CASE STUDY BASED QUESTIONS

I.   Read the following passage and choose appropriate answers of questions 1 to 4.

The rotational analogue of force in linear motion is moment of force. It is also referred to as torque or couple. If a force acts on a single particle at a point, whose position with respect to the origin is given by the position vector r, the moment of the force acting on the particle with respect to the origin is defined as the vector product

$$\tau = r \times F$$

The moment of force (or torque) is a vector quantity.
The magnitude of is

$$\tau = r\,F\,\sin\theta$$

Where $r\sin\theta$ is the perpendicular distance of the line of action of F from the origin and
$F\sin\theta$ is the component of F in the direction perpendicular to r. Note that $\tau = 0$ if $r = 0$,
$F = 0$ or $\theta = 0^0$ or 180^0 .
Thus, the moment of a force vanishes if either the magnitude of the force is zero, or if the line of action of the force passes through the origin.
With the help of above comprehension, choose the most appropriate alternative for each of the following questions:

1. If directions of both r and F are reversed, the direction of the moment of force
    a. remains the same.
    b. Reverse in direction
    c. Becomes parallel to force applied.
    d. Becomes parallel to position vector
2. The dimensional formula of torque is same as that of
    a. Angular momentum
    b. Work
    c. Momentum
    d. Force
3. Torque is maximum when the angle between F and r is
    a. 0^0
    b. 180^0
    c. 90^0
    d. 360^0
4. Wrench of longer arm is preferred because
    a. It produces maximum force
    b. It produces maximum torque.
    c. It is easy to hold
    d. Wrench of shorter arm is equally good.

II.   Read the following passage and choose appropriate answers of questions 5 to 8.

The centre of mass of a body is a point at which the entire mass of the body is supposed to be concentrated. The position vector $\vec{r}$ of C.O.M. of the system of two particles of masses m_1 and m_2 with position vectors $\vec{r1}$ and $\vec{r2}$ is given by

$$\vec{r} = \frac{m_1\vec{r1} + m_2\vec{r2}}{m_1 + m_2}$$

For isolated system, where no external force is acting, V_{cm} = constant

Under no circumstances, the velocity of the C.O.M. of an isolated system can undergo a change

With the help of above comprehension, choose the most appropriate alternative for each of the following questions:

5. Two bodies of masses 1kg and 2kg are located at (1,2) and(-1,3) respectively . The co-coordinates of C.O.M.are

    (a)   (-1, 3)

    (b)   (1, 2)

    (c) $(-\frac{1}{3},\frac{8}{3})$

    (d) $(\frac{1}{3},-\frac{8}{3})$

6. Two blocks of masses 5kg and 2kg are placed on a frictionless surface and connected by a spring .An external kick gives a velocity of 14m/s to heavier block in the direction of lighter one. The velocity gained by the C.O.M. is

    (a)   14m/s

    (b)   7m/s

    (c)   12m/s

    (d)   10m/s

7. An electron and proton of an atom move towards each other with velocities v_1 and v_2 respectively. The velocity of their Centre of mass is

    (a)   zero

    (b) v_1

    (c) v_2

    (d) $\frac{v1+v2}{2}$

8. A bomb dropped from an aeroplane in level flight explodes in the middle. The centre of mass of the fragments

    (a)   is at rest

    (b)   Moves vertically downwards

    (c)   Moves vertically upwards

    (d)   continues to follow the same parabolic path which it would have followed if there was no explosion.

III.   Read the following passage and choose appropriate answers of questions 9 to 12.

Moment of inertia of a body about a given axis is the rotational inertia of the body about that axis. It is represented by I= MK^2 , where M is mass of body and K is radius of gyration of the body about that axis . It is a scalar quantity , which is measured in kgm^2 . When a body rotates about a given axis , and the axis of rotation also moves , then total K.E.of body =K.E. of translation + K.E. of rotation

$$E = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

With the help of above comprehension, choose the most appropriate alternative for each of the following questions:

9. Moment of inertia of a body depends on

   (i) mass of body (ii) size and shape of body (iii) axis of rotation of body (iv) all the above

   (a)   (i) and (ii)

   (b)   (i) and (iii)

   (c)   (ii) and (iii)

   (d)   (iv)

10. A circular disc and a circular ring of same mass and same diameter have _____about a given axis .

    (a) same moment of inertia

    (b) unequal moments of inertia

    (c) cannot say

    (d) sometimes equal sometimes not

11. A 40kg flywheel in the form of a uniform circular disc of diameter 1m is making 120 rpm . Its moment of inertia about a transverse axis through its centre is

    (a) 40kgm^2

    (b) 5kgm^2

    (c) 10kgm^2

    (d) 20kgm^2

12. Kinetic energy of rotation of flywheel in the above case is

    (a)   20J

    (b)   2J

    (c)   400J

    (d)   80J

IV.   Read the following passage and choose appropriate answers of questions 13 to 16.

The time rate of the total angular momentum of a system of particles about a point (taken as the origin of our frame of reference) is equal to the sum of the external torques (i.e. the torques due to external forces) acting on the system taken about the same point.

$$\tau_{ext} = \frac{dL}{dt}$$
$$\text{If } \tau_{ext} = 0$$
$$\frac{dL}{dt} = 0$$
or L = constant.
Or $I\omega = constant$

Thus, if the total external torque on a systemof particles is zero, then the total angularmomentum of the system is conserved, i.e.remains constant.

With the help of above comprehension, choose the most appropriate alternative for each of the following questions:

13. Which of the following can be explained with the help of conservation of angular momentum?
    a. Driving
    b. Ice- skating
    c. Diving
    d. running

14. For angular momentum to be conserved what must be true about the net torque of the system?

    a. Net torque is constant.
    b. Net torque increases.
    c. Net torque decreases.
    d. Net torque is zero.

15. A person sits on a freely spinning lab stool that has no friction in its axle. When this person extends her arms,
    a. her moment of inertia increases and her angular speed decreases.
    b. her moment of inertia decreases and her angular speed increases.
    c. her moment of inertia increases and her angular speed increases.
    d. her moment of inertia increases and her angular speed remains the same.

16. Two children, Ahmed and Saleh, ride on a merry-go-round. Ahmed is at a greater distance from the axis of rotation than Saleh. Which of the following are true statements?
    a. Saleh and Ahmed have the same tangential speed.
    b. Ahmed has a greater tangential speed than Saleh.
    c. Saleh has a greater angular speed than Ahmed.
    d. Saleh has a smaller angular speed than Ahmed

V.   Read the following passage and choose appropriate answers of questions 17 to 20.

The forces change the translational state of the motion of the rigid body, i.e. they change its total linear momentum. But this is not the only effect the forces have. The total torque on the body may not vanish. Such a torque changes the rotational state of motion of the rigid body, i.e. it changes the total angular momentum of the body.

A rigid body is said to be in mechanicalequilibrium, if both its linear momentum and angular momentum are not changing with time, or equivalently, the body has neither linear acceleration nor angular acceleration. This means

(1) The total force, i.e. the vector sum of the forces, on the rigid body is zero;
$$\sum F = 0$$
If the total force on the body is zero, then the total linear momentum of the body does not change with time. This gives the condition for the translational equilibrium of the body.

(2) The total torque, i.e. the vector sum of the torques on the rigid body is zero,
$$\sum \tau = 0$$
If the total torque on the rigid body is zero, the total angular momentum of the body does not change with time. This gives the condition for the rotational equilibrium of the body.

With the help of above comprehension, choose the most appropriate alternative for each of the following questions:

17. Which of the following is NOT a condition for an object to be in static equilibrium?
a. The object is not moving.
b. It is in translational equilibrium.
c. It is in rotational equilibrium.

d. It is moving with constant velocity.

18. The easiest way to open a heavy door is by applying the force
    a. Near the hinges
    b. In the middle of the door
    c. At the edge of the door far from the hinges
    d. Anywhere on the door.

19. If a system is in translational equilibrium, it must have the conditions:

    a. $\sum M = 0$
    b. Constant acceleration
    c. $\sum F = 0$
    d. Positive forces only

20. The moment of a 50 N force 20 cm away from the pivot point will be...
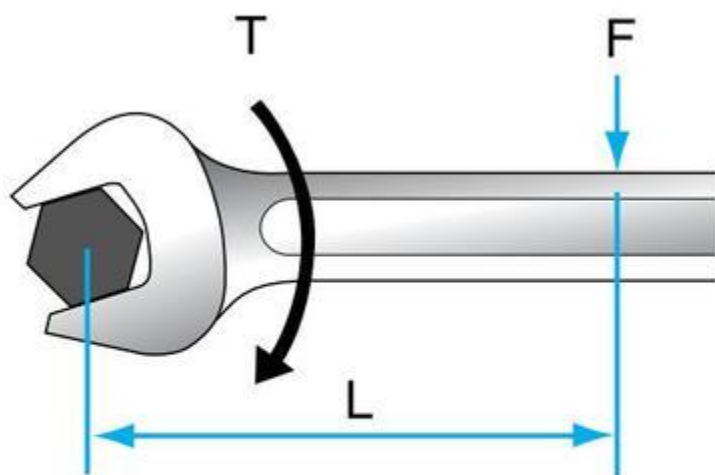
Torque T = F (Force) × L (Length)

    a. 500 N
    b. 20 Nm
    c. 10 Nm
    d. 15 Nm

ANSWER KEY

ANSWERS OF MULTIPLE CHOICE QUESTIONS

1 (c) L/2√3

2. (a) 2.5 v

3. (a) The vector is perpendicular to the orbital plane

4. (b) Less than g sin Θ

5. (b) Angular velocity

6. (c) There is a conversion of translational motion into rotational motion

7. (d) Angular momentum

8. (d) L/4

9. (c) Sphere

10. (d) The rotational kinetic energy of a particle

11. (c) Angular momentum

12. (a) When the motion is rotational

13. (a) 1 m/s

14. (b) L/4

15. (a) g/2

16. (d) $\frac{1}{2}I\omega^2$

17. (a) $\frac{1}{2}MR^2$

18. (d) Axis of rotation

19. (a) 2.5

20. (d) The direction of angular momentum remains constant.

21. (b) Moment of inertia

22. (d) I = τa

23. (b) Angular momentum

24. (d) angular momentum

25. (c) Torque

26. (d) $\pi/1800$ rad s^{-1}

27. (b) 2:1

28. (a) t = Ia

29. (c) Constant angular momentum and increase in kinetic energy

30. (c) (1/2) Mr²

31. (c)$\hat{\imath} \times \hat{\jmath} = \hat{k}$

32. (d) 21%

33. (a) pure rotation

34. d

35. b

36. a

37. d

38. a

39. c

40. d

41. a

42. d

43. d

44. d

45. c

46. c

47. a

48. b

49. a

50. b

ANSWERS OF ASSERTION AND REASONING QUESTIONS

1. a

2. b

3. c

4. d

5. d

6. b

7. d

8. d

9. d

10. a

11. a

12. a

13. b

14. c

15. b

16. a

17. c

18. c

19. a

20. a

ANSWERS OF CASE STUDY BASED QUESTIONS

1. A
2. b
3. c
4. c
5. c
6. d
7. a
8. d
9. d
10. b
11. b
12. c
13. c
14. d
15. a
16. b
17. a
18. c
19. c
20. c